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AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by J. H. CURTISS

VOLUME 6, NUMBER 3 ISSUE NO. 31 JUNE 1958

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Please send in abstracts of papers to be presented in person well in advance of the deadline.

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
548	August 26-29, 1958 (63rd Summer Meeting)	Cambridge, Massachusetts	July 11
549	October 25, 1958	Princeton, New Jersey	Sept. 11
550	November 21-22, 1958	Pomona, California	Oct. 8
551	November 28-29, 1958	Durham, North Carolina	Oct. 8
552	November 28-29, 1958	Evanston, Illinois	Oct. 8
553	January 20-22, 1959 (65th Annual Meeting)	Philadelphia, Pennsylvania	Dec. 5
	Summer Meeting, 1959	Salt Lake City, Utah	
	November, 1959	Detroit, Michigan	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

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FIVE HUNDRED FORTY-SEVENTH MEETING

Corvallis, Oregon
June 21, 1958

PROGRAM

The five hundred forty-seventh meeting of the American Mathematical Society will be held on Saturday, June 21, 1958, at Oregon State College in Corvallis, Oregon. There will be a meeting of the Mathematical Association of America on Friday, June 20, and a meeting of the Society for Industrial and Applied Mathematics on Saturday afternoon.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, the Society will be addressed on "Problems on Riemann surfaces" by Professor Leo Sario of the University of California, Los Angeles, at 10:00 A.M. Saturday in the Withycombe Hall auditorium, Room 101. Sessions for contributed papers will be held at 1:30 P.M. on Saturday in Agriculture Hall, Rooms 132,138, and 140. Late papers can be accommodated, details to be available at the registration desk in Sackett Hall C (or at the door of Withycombe 101).

Dormitory accommodations will be available at $2.00 per person per night, beginning Thursday evening June 19, and extending through Saturday night, June 21, at Sackett Hall C. Food service will be provided at the Memorial Union Coffee Shop from breakfast on June 20 to dinner on June 21. There will be an MAA dinner Friday evening, June 20, at the Memorial Union Tearoom, costing $2.00. Reservations are to be sent to Professor R. D. Stalley, Department of Mathematics, Oregon State College, Corvallis, Oregon, by June 13, stating time of arrival, dormitory accommodations required, and number of reservations for the dinner Friday evening.

For those desiring other accommodations, the following are listed. Reservations should be made directly to the hotel or motel.

	Single	Double
Benton Hotel	$4.00 - 5.00	$5.50 - 6.50
City Center Motel	5.00	7.00
Don Mar Hotel	5.00	7.00

Corvallis is about 80 miles south of Portland on US Highway 99W. It is served by West Coast Airlines and by Greyhound Bus. The Southern Pacific railroad line passes through Albany, ten miles east of Corvallis, and Greyhound service is available from Albany to Cor-

vallis. (However, connections between Southern Pacific and Greyhound are less than ideal.)

Program details at the meeting will be handled by Professor R. S. Pierce of the University of Washington, who will be Acting Associate Secretary from June 1958 to September 1959.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

SATURDAY, 10:00 A.M.

General Session, Withycombe Hall, Room 101

Problems on Riemann surfaces (One hour)

Professor Leo Sario, University of California, Los Angeles

SATURDAY, 1:30 P.M.

Session on Algebra, Number Theory, Foundations, and Topology, Agriculture Hall, Room 132

(1) A Diophantine problem associated with the homogeneous product sum
Professor Morgan Ward, California Institute of Technology (547-16)

(2) Diagonals of doubly stochastic matrices
Professor M. D. Marcus and Dr. Rimhak Ree, University of British Columbia (547-8)

(3) On a problem of Kurepa concerning the axiom of choice
Professor Herman Rubin, University of Oregon (547-11)

(4) Translation lattices. I
Professor R. S. Pierce, University of Washington (547-1)

(5) Convolutions semigroups of measures
Professor I. L. Glicksberg, University of Notre Dame (547-15)

(6) A class of function algebras
Professor F. W. Anderson, University of Oregon (547-14)

(7) Spaces of real ideals
Professor F. W. Anderson and Professor R. L. Blair, University of Oregon (547-13)

(8) Fixed point theorems for arcwise connected spaces
Dr. L. E. Ward, Jr., U. S. Naval Ordnance Test Station, China Lake (547-4)

Session on Geometry and Analysis, Agriculture Hall, Room 138

(9) Covariant differentiation
Mr. B. L. Foster, University of Wisconsin (547-12)

(10) The zeros of some sine-integral-like functions
Professor A. E. Livingston, University of Washington, and Professor Lee Lorch, Philander Smith College (547-9)

(11) A new class of continued fraction expansions for the ratios of hypergeometric functions. II
Professor Evelyn Frank, University of Illinois (547-17)

(12) Concerning the Hausdorff inclusion problem
Mr. J. H. Wells, University of Texas (547-3)

(13) Integral curves of a certain class of differential systems
Dr. C. S. Coleman, Wesleyan University (547-2)

Session on Analysis and Applied Mathematics, Agriculture Hall, Room 140

(14) An alternating gradient method for the solution of linear systems
Mr. E. C. Riggle, Oregon State College (547-18)
(Introduced by Professor H. E. Goheen)

(15) An iterative solution of the quadratic equation in Banach space
Mr. J. E. McFarland, Oregon State College (547-19)

(16) Linear integral equations for the generating functions of a class of functionals of Markov processes
Dr. R. B. Leipnik, U. S. Naval Ordnance Test Station, China Lake (547-10)

(17) Application of the Fréchet differential to the approximate solution of the Volterra integral equation
Mr. John Hilzman, Oregon State College (547-5)
(Introduced by Professor A. T. Lonseth)

(18) On the Fredholm alternative in locally convex spaces
Professor H. H. Schaefer, State College of Washington (547-6)
(Introduced by Professor S. G. Hacker)

(19) Asymptotic perturbation series for characteristic value problems
Dr. C. A. Swanson, University of British Columbia (547-7)

SUPPLEMENTARY PROGRAM
(To be presented by title)

(20) An implicit numerical method for solving the two-dimensional heat equation
Dr. G. A. Baker, Jr. and Mr. T. A. Oliphant, Los Alamos Scientific Laboratory
(Introduced by Dr. T. C. Doyle)

(21) Application of group representation theory to determination of natural frequencies of vibrating plates
Mr. K. L. Brinkman, Hughes Research and Development Laboratories, Culver City, California, and Mr. L. I. Deverall, University of California Radiation Laboratory, Livermore

(22) Loop spaces of suspensions. II
Mr. William Browder, University of Rochester

(23) On the EPD equation and subharmonic distributions. Preliminary report
Mr. R. W. Carroll, University of Maryland

(24) Some remarks on Galois theory
Professor Eckford Cohen, University of Tennessee

(25) Arithmetical inversion formulas
Professor Eckford Cohen, University of Tennessee

(26) Stable conal regions in the neighborhood of a singular point in three space
Professor C. S. Coleman, Wesleyan University

(27) On sets with positive reach, applied to curvature theory. Preliminary report
Professor Herbert Federer, Brown University

(28) On the numerical solution of Poisson's equation with given boundary values
Professor Donald Greenspan, Purdue University

(29) A sufficient condition that a monotone image of the three-sphere be a topological three-sphere
Professor O. G. Harrold, Jr., University of Tennessee and Institute for Advanced Study

(30) Algebraic extensions of Banach algebras
Professor G. A. Heuer, University of Minnesota and Concordia College

(31) Singularities of solutions of partial differential equations in three real variables
Professor E. O. A. Kreyszig, Ohio State University

(32) On the self-intersection of an immersed manifold
Professor R. K. Lashof and Dr. Stephen Smale, University of Chicago

(33) A semantic Herbrand-Gentzen lemma
Professor R. C. Lyndon, University of Michigan

(34) On the cohomology ring of a sphere bundle
Mr. D. G. Malm, Rutgers University

(35) A canonical form for antiderivatives
Professor E. J. McShane, University of Virginia

(36) The axiom of fundierung and the axiom of choice
Dr. Elliott Mendelson, Harvard University

(37) On the cobordism ring Ω^*
Professor J. W. Milnor, Princeton University

(38) Elliptic differential equations with Holder-continuous coefficients
Professor C. B. Morrey, Jr., University of California, Berkeley

(39) Finite groups. Coset decompositions
Dr. Elvira S. Rapaport, Stockbridge, Massachusetts

(40) Finite groups. Graphs
Dr. Elvira S. Rapaport, Stockbridge, Massachusetts

(41) Unified interpretation of logical systems
Mr. D. E. Schroer, University of Rochester

(42) On centered difference equations for hyperbolic systems
Dr. Burton Wendroff, Los Alamos Scientific Laboratory

(43) On the invariance of openness
Professor G. T. Whyburn, University of Virginia

(44) Compactness of certain mappings
Professor G. T. Whyburn, University of Virginia

(45) Disturbance of a many-dimensional field satisfying the Helmholtz equation due to the presence of a hyperplane boundary
Dr. G. C. K. Yeh and Mr. Johann Martinek, Reed Research Inc., Washington, D. C.

V. L. Klee, Jr.
Associate Secretary

Seattle, Washington
March 8, 1958

PRELIMINARY ANNOUNCEMENT OF MEETING

SIXTY-THIRD SUMMER MEETING

Massachusetts Institute of Technology

Cambridge, Massachusetts
August 26-29, 1958

The sixty-third summer meeting of the American Mathematical Society will be held from Tuesday, August 26, through Friday, August 29, 1958, at the Massachusetts Institute of Technology, Cambridge, Massachusetts, in conjunction with meetings of other organizations as follows: Mathematical Association of America from Monday, August 25, through Thursday, August 28; Institute of Mathematical Statistics from Monday, August 25, through Thursday, August 28; Society for Industrial and Applied Mathematics from Monday, August 25, through Wednesday, August 27; and the Econometric Society. Preliminary announcements of the meetings of some of these organizations appear elsewhere in this issue of the NOTICES.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, five hour addresses will be given. Professor B. J. Pettis of the University of North Carolina will address the Society at 2:00 P.M. on Tuesday on "The existence and extension measures". Professor Eldon Dyer of the University of Chicago will deliver an address entitled "The effect of mappings on dimension" at 9:00 A.M. on Wednesday. On Thursday there will be two invited addresses: Professor Walter Rudin of the University of Rochester will speak at 9:00 A.M. on "Measure algebras on abelian groups", and at 10:30 A.M. Professor O.M. Nikodym of Kenyon College will deliver an address entitled "Mathematically precise setting of the genuine Dirac's δ-function and proof of its basic properties". Professor Jürgen Moser of the Massachusetts Institute of Technology will address the Society at 9:00 A.M. on Friday. All invited addresses will be given in Kresge Auditorium.

Since this meeting will be almost concurrent with the International Congress of Mathematicians to be held in Edinburgh, Scotland, no Colloquium Lectures are scheduled.

Sessions for contributed papers will be held each afternoon, Tuesday through Friday, and on Friday morning.

There will be a business meeting of the Society in Kresge Auditorium at 10:15 A.M. on Wednesday.

The Council of the Society will meet at 5:00 P.M. on Tuesday.

The Employment Register will be maintained near the registration desk. The Providence office of the Society is now handling all of the details for the Employment Register. Listings for this meeting should be sent to 190 Hope Street, Providence 6, Rhode Island, or brought to the meeting.

REGISTRATION

Registration headquarters will be in the lobby of Kresge Auditorium, and will be open Sunday afternoon and evening, August 24, from 2:00 to 10:00 P.M. and from 8:30 A.M. to 5:00 P.M., Monday through Friday. All those attending the meetings are requested to register as soon as possible. Those arriving after the registration desk is closed, and having reservations, may go directly to Baker House. A directory of all persons registered, and an information desk will be available in the registration area. The text book exhibits will be located in Baker House.

There will be a registration fee of $2.00 for each member of any participating organization and $1.00 for each accompanying adult. Tickets for the beer party will cost $1.00 per person.

ROOMS AND MEALS

Dormitory accommodations are available. Walker Memorial near the dormitories and meeting places will be open especially for these meetings for a light breakfast a la carte, from 7:30 to 9:30 A.M. and a set $ .99 lunch, from 11:30 to 1:30, from Monday breakfast through Friday noon. Tickets for lunch may be obtained at the registration desk and at the cafeteria. Supper may be had at one of several nearby restaurants or at the M. I. T. Graduate House Dining Room, from 5:00 to 7:00 P.M. Saturday breakfast may also be obtained at the Graduate House from 9:00 to 11:00 A.M. Dormitory rooms may be occupied from Sunday evening, August 24, until the following Saturday. A reservation form is to be found on page 385; members are requested to fill out the form and return it to Professor Philip Franklin, 2-163, M. I. T., Cambridge 39, Massachusetts, in time to arrive by August 9, regardless of whether or not they wish dormitory reservations. (Inasmuch as the August NOTICES will not reach the membership in time to meet this deadline of August 9, MEMBERS ARE ASKED TO USE THE RESERVATION FORM ON PAGE 385 OF THIS ISSUE OF THE NOTICES IN ORDER TO MAKE RESERVATIONS).

Many double rooms and a limited number of single rooms in the M. I. T. dormitory system are available.

For single men or women, the housing rate will be $3.50 per person for each twenty-four hour unit beginning at 6:00 P.M.; for married couples $2.50 per person; and for children 16 and under

$2.00 per person. [For children under 12 who wish to be in the same room as their parents, M. I. T. will make available cots at a rate of $1.00 per person].

Some of the Boston and Cambridge hotels which are conveniently located with respect to M. I. T. are listed below. A few of the accommodations for motorists within commuting distance are also listed. Since the number of rooms in these motels is relatively small, it is suggested that those desiring such accommodations make their reservations well ahead of time.

HOTELS

Boston		Rates per person *
Kenmore Hotel KEnmore 6-2770	490 Commonwealth Ave. at Kenmore Square	$6.00 up
Ritz-Carlton Hotel KEnmore 6-5700	Arlington and Newbury Sts.	9.00 up
Sheraton Plaza Hotel COpley 7-5300	138 St. James Ave. in Copley Square	6.85 up
Sherry Biltmore Hotel COpley 7-7700	150 Massachusetts Ave.	5.00 up
Somerset Hotel KEnmore 6-2700	400 Commonwealth Ave.	7.00 up
Statler Hotel HAncock 6-2000	Arlington Street at Park Square	6.50 up
Cambridge (near Harvard Square)		
Ambassador Hotel UNiversity 4-6200	1737 Cambridge Street	6.00 up
Commander Hotel KIrkland 7-4800	16 Garden Street	6.00 up
Continental Hotel KIrkland 7-6100	29 Garden Street	5.00 up

MOTELS AND MOTOR COURT HOTELS

Boston	Rates*	Single	Double
Hampton Court Hotel BEacon 2-7500	1223 Beacon Street Brookline, Mass. Kitchenettes, family rates, parking.	$9.50 up	$14.00 up

* Rates are subject to change

1200 Beacon Street	1200 Beacon Street Brookline, Mass. No charge for children under 14 in the same room.	$10.50 up	$15.00 up
Terrace Motel Apts. LOngwood 6-6260	1650 Commonwealth Ave. Boston 35, Mass. Airconditioned	7.00 up	10.00 up
Charterhouse Motel LAsell 7-9000	On State Rte. 9 at Chestnut Hill	8.80 up Family - $16.60 No Kitchenettes	13.30 up
Bedford			
Bedford Motel CRestview 4-6300	South Edge of Bedford on State Rte. 4	7.00 up	9.00 up
Concord			
The Concordian Motel COlonial 3-7765	4 miles west of Concord on State Rte. 2 P. O. Acton	7.00 up	10.00 up
Colonial Inn EMerson 9-4600	Monument Square in Concord. On State Rte. 2A	7.00 up	10.00 up
Framingham Center			
Framingham Motor Inn TRinity 2-1206	1 1/2 miles west of Fram- ingham on State Rte. 9	8.50 up	

ENTERTAINMENT AND RECREATION

Lounges - Rehearsal Room A on the lower floor of the Kresge Auditorium will be open daily including Sunday, from 8:00 A. M. to 11:00 P. M.; the Mathematics Common Room (Room 2-290) will be open from 9:00 A. M. to 5:00 P.M.Monday through Friday; the lounge in Baker House will be available at all times.

The Alumni Pool will be open for swimming Monday through Friday; on Wednesday from noon to 6:30 P.M. and for women only from 6:30 to 8:00 P.M.; and otherwise from noon to 8:00 P.M.; suits, towels and lockers will be provided. Women should provide their own caps. Those interested in sailing in the M. I. T. dinghies on the Charles River must first obtain from the swimming pool a certificate of swimming ability. The du Pont tennis courts on Briggs Field are open from 8:00 A.M. to dusk and may be reserved by telephone or at the courts for not more than one hour a day. Badminton and volley-

ball courts in the Armory are open for play from 9:00 A.M. to 6:00 P.M.; lockers and the equipment required for softball, badminton and volleyball will be supplied on request. Registrants will be issued an Athletic Privilege Card which will provide admission without charge to any of these recreational facilities. Telephone: Tennis courts--Ext. 2912; Briggs Field--Ext. 2913.

The annual SIAM beer party, open by ticket to all registrants, will be held in the Walker Memorial Building on the M. I. T. Campus on Tuesday evening at 8:30. Admission is $1.00 per person; tickets available at the registration desk in advance.

The M. I. T. Administration cordially invites all registrants to a tea and reception to be held Wednesday afternoon, 4:00 to 5:30 P.M., in the President's House, 111 Memorial Drive.

A list of baby sitters will be available at the registration desk.

Information with respect to special events and points of interest on the campus and in and about Greater Boston, and a list of restaurants, will be available at the registration desk.

TRANSPORTATION

M. I. T. is located on the Charles River opposite the Back Bay region of Boston. The main building is at 77 Massachusetts Avenue, Cambridge. Registration headquarters will be in the lobby of Kresge Auditorium, distinguished by its spherical triangular dome, and located directly across Massachusetts Avenue from the main building.

Boston and Cambridge are on U. S. Rtes. 1, 3, and 20; and Mass. State Rtes. 2, 9, 16, 28, 30, and 38. The East-West Mass. Turnpike ends at State Rtes. 128 and 30 on the outskirts of Boston. Follow Rte. 30 Eastbound along Commonwealth Avenue to the intersection with U. S. Rte. 1 at the Boston University Bridge; then U. S. Rte. 1 Northbound across the river and to M. I. T. at the next bridge which is on Massachusetts Avenue.

Boston is served by Greyhound, Vermont Transit, and Trailways Buses. The terminals are located in Park Square near the Arlington Street Subway entrance. From the Arlington Street subway station take Beacon Street or Commonwealth Avenue car to Massachusetts Station, then Harvard Square bus to M. I. T. Taxi fare to M. I. T. from Park Square is approximately $1.25.

Rail passengers arriving at South Station or North Station may take the subway to Park Street, thence southbound to Massachusetts Station, thence Harvard Square Bus to M. I. T. From Back Bay Railroad station, taxi fare to M. I. T. is approximately $1.

Airline passengers arriving at the Logan Airport in Boston may use public bus and subway transportation (20 cents fare) as follows: MTA bus from airlines terminal to Airport subway station, subway to Scollay Square, then Beacon Street or Commonwealth Avenue car to Massachusetts Station, transfer there to Harvard Square Bus and get off at M. I. T.

MAIL AND TELEGRAMS

Correspondence for those attending the meetings should be addressed in care of the American Mathematical Society, Kresge Auditorium, M. I. T., Cambridge 39, Massachusetts. Mail may be obtained from the mail desk in the registration area.

Committee on Arrangements:

J. H. Curtiss	Norman Levinson
Philip Franklin	Hartley Rogers, Jr.
H. M. Gehman	R. D. Schafer
F. B. Hildebrand	G. B. Thomas, Jr., Chairman

Further details of the meeting will appear in the next issue of the NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, July 11. It is expected that abstracts of all papers to be presented in person will appear in the same issue of the NOTICES, but only abstracts which meet the specifications stated on the abstract blanks can be published. Massachusetts Institute of Technology will provide audio-visual aids (such as slide projector with operator, etc.) for those authors desiring them; requests should accompany the abstracts.

R. D. Schafer
Associate Secretary

Storrs, Connecticut
May 7, 1958

ACTIVITIES OF OTHER ASSOCIATIONS

INTERNATIONAL MATHEMATICAL UNION. Dr. B. Eckmann, Secretary of the International Mathematical Union, has announced that the application of Rumania for membership in the International Mathematical Union (Group II) has been unanimously approved by the voting nations. The membership of Rumania became effective on March 1, 1958.

THE MATHEMATICAL ASSOCIATION OF AMERICA. The thirty-ninth summer meeting of the Mathematical Association of America will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, from Monday, August 25 to Thursday, August 28, 1958. The following is a preliminary announcement of the program:

Five sessions of the Association will be held on Monday morning and afternoon, Tuesday morning, Wednesday afternoon and Thursday afternoon. At the first three sessions Dr. Alston S. Householder of Oak Ridge National Laboratory will deliver the seventh series of Earle Raymond Hedrick Lectures on the general topic of "Some mathematical problems arising in computations with matrices".

The general theme of the meeting of the Association will be Applied Mathematics. It is expected that the following topics will be discussed:

Mathematical physics
Probability and statistics
Numerical analysis and computing machines
Theory of games and mathematical analysis

A meeting of the Board of Governors will be held on Monday evening and a meeting of Section Officers of the Association will be held on Tuesday evening.

THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS. The 1958 summer meeting of the Society for Industrial and Applied Mathematics will be held at the Massachusetts Institute of Technology on Monday, Tuesday, and Wednesday, August 25 through 27. The sessions are being held in conjunction with the American Mathematical Society and the Mathematical Association of America.

The SIAM meetings will feature invited lectures by Professor Stephen H. Crandall, Mechanical Engineering Department, MIT, Dr. Irving Gerst, Control Instrument Company, and Dr. Harlan D. Mills, Princeton University.

Several sessions on contributed papers are being arranged by the Program Committee. Contributed papers should be in some field of applied mathematics such as computer theory, network theory, stress analysis, nucleonics, probability theory, communications

theory, hydrodynamics, nerve network theory, mathematical programming, or systems analysis. Abstracts should be directed to a member of the Program Committee no later than July 11, 1958. Time for presentation is limited to fifteen minutes.

Present plans call for a beer party on the evening of Tuesday, August 26.

Members of the SIAM Program Committee are:

Dr. H. M. Gurk - Mathematical Analyst
Radio Corporation of America
Camden, New Jersey

Dr. Brockway McMillan
Bell Telephone Laboratories, Inc.
463 West Street
New York, New York

Dr. D. L. Thomsen, Jr., Chairman
IBM Corporation
425 Park Avenue
New York 22, New York

THE FRENCH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCES will hold its 77th Congress at Namur, Belgium, on the 15th to the 20th of July, 1958. The President of the Congress will be J. Pérès. The mathematical section is headed by L. Godeaux, the subsection on Numerical Analysis is under P. Vernotte, and the section of Mechanics is under A. Martinot-Lagarde.

A CONFERENCE ON MATHEMATICAL STATISTICS TO BE HELD AT THE UNIVERSITY OF ST. ANDREWS IN SCOTLAND. The Research Section and the Industrial Applications Section of the Royal Statistical Society intend to hold a Conference at the University of St. Andrews, near Edinburgh, Scotland, from 22 August to the 25 August inclusive. It will be devoted to Mathematical Statistics, with special reference to those branches of the subject which have application in industry.

It is proposed that there should be three morning sessions, consisting each of two or three pre-arranged lectures, and three early evening sessions (5:30 P.M. to 6:30 P.M.) each with one pre-arranged lecture.

The afternoons will be devoted to 'splinter groups' which will devote themselves to special aspects, and at which informal talks of some ten or fifteen minutes each can be given without prior arrangement.

Topics to be covered in the morning and evening sessions include aspects of the analysis of variance, non-parametric inference, stochastic aspects of linear and dynamic programming, and foundations

of probability in statistics.

It is hoped that many of the mathematical statisticians who will be coming from abroad to attend the Edinburgh International Congress of Mathematicians, will choose to remain in Scotland for a further few days, and take the opportunity of meeting colleagues specially interested in their field. St. Andrews, besides having the famous Golf Course, is a small Scottish town of considerable character, and a very good centre for the exploration of the Eastern Highlands.

Accommodation (from 21 August to 25 August) will be provided within the hostels of the University of St. Andrews at a reasonably low cost, details to follow later. Anyone interested should write, marking the envelope 'ST. ANDREWS CONFERENCE' to Miss U. Croker, Royal Statistical Society, 21 Bertinck St., London, W. I., England.

SECOND INTERNATIONAL CONGRESS ON CYBERNETICS. The International Association for Cybernetics is organizing the Second International Congress on Cybernetics in Namur, Belgium, from September 3 to September 10, 1958. The purpose of the Congress is to bring together people with a common interest in cybernetics, thus continuing the work of the 1956 Congress which was attended by 800 delegates from 22 countries. The organizers hope for the widest possible exchange of views and information.

The subjects dealt with at the Congress will be divided into the following six groups:

1. Information (information theory, computing machinery, etc.).
2. Automatic machinery (the application of cybernetics to machines).
3. Automation (the use of automatic machinery in industry and industry and office work).
4. The economic and social effects of automation.
5. Cybernetics and the social sciences.
6. Cybernetics and biology.

The languages used at the Congress will be French and English. The program will include plenary sessions and meetings of the various sections. A concluding conference will synthesize the general results of the Congress. The plenary sessions will be held in the mornings, and at these sessions general lectures will be given on the subjects mentioned above. The section meetings will be held in the afternoons. At these meetings papers will be read and discussed. There will be as many sections as necessary to cover the maximum amount of ground. The concluding conference of the Congress will be attended by the chairmen and secretaries of the sections. It is expected that the proceedings of the Congress will be published.

The author of each paper to be presented is requested to send the title and a summary of the paper to the Secretariat of the International Association for Cybernetics, 13, rue Basse-Marcelle, Namur, Belgium, as soon as possible and in no case later than June 1, 1958.* This summary should not exceed one typewritten page.

A visit to the 1958 Brussels World's Fair will be arranged for the members of the Congress and their families. In addition, a series of excursions will be organized for the persons accompanying the participants but not attending the meetings.

Those who wish to take part in the Congress are asked to fill in an application form which can be obtained from the Secretariat of the International Association for Cybernetics at the above address. The fees for attending the Congress have been fixed as follows:

Members of the International Association for Cybernetics . . . . . 200 francs
Non-members . . . . . 400 francs
No fees will be charged to the authors of papers.

The Organizing Committee of the Congress is as follows:

President: Professor Georges R. Boulanger (Belgium), of the Polytechnic Faculty of Mons and of Brussels University.

Members: René Close (Belgium), Barrister at Namur;
Louis Couffignal (France), Inspector-General of Education, Director of the Automatic Computation Laboratory, Institut Blaise Pascal, Paris;
John Diebold (U.S.A.), Management Consultant, President of John Diebold and Associates, Inc., New York;
W. Grey Walter (United Kingdom), Director of the Physiological Department of the Burden Neurological Institute, Bristol.

Permanent Delegate: Josse Lemaire (Belgium), Director of the Economic, Social, and Cultural Office of the Province of Namur.

*EDITORIAL NOTE: We regret that this announcement was not received in this office in time to publish in an earlier issue of the NOTICES.

NEWS ITEMS AND ANNOUNCEMENTS

APPLICATIONS FOR GRADUATE FELLOWSHIPS IN FIELDS OF RESEARCH UTILIZING A DIGITAL COMPUTER are invited by the Computations Centre, McLennan Laboratory, The University of Toronto. Applicants need not be Canadian citizens. Applications should be sent to the Computations Centre, marked "Fellowship".

A SUMMER INSTITUTE FOR HIGH SCHOOL TEACHERS OF MATHEMATICS TO BE HELD AT FORDHAM UNIVERSITY. With the support of the National Science Foundation, Fordham University is conducting a Summer Institute for High School Teachers of Mathematics from July 7 to August 14 this summer. The Institute offers courses in Probability and Statistical Inference, and Basic Concepts and Structures of Geometry. Graduate credit is obtainable in the School of Education. Alternates selected by other NSF sponsored Summer Institutes have been invited to apply to the Fordham Institute. Further information may be obtained from the Reverend Charles J. Lewis, S. J., Fordham University, New York 58, New York.

THE CANADIAN CONFERENCE FOR COMPUTING AND DATA PROCESSING will be held at the University of Toronto on June 9 and 10, 1958. This conference is organized in the public interest as a self-supporting education venture. The sponsors are the University of Toronto and a group of companies and organizations who are using or will shortly be using computers. The conference will be financed entirely by the registration fees which are $35.00 per person or $100 for three persons. These fees include attendance at sessions, and an evening reception and banquet at the King Edward Hotel in Toronto, luncheons on both days, and a complete copy of the conference proceedings including a full reprint of the papers. Reprints will be available to registrants only. Further information can be obtained from Mr. H. W. Rowlands, 15 Wellington Street, W., Toronto, Ontario.

A MEETING ON HYDRODYNAMICS, SPONSORED BY THE LOS ALAMOS SCIENTIFIC LABORATORY, is to be held on October 16-18, 1958 at Los Alamos, New Mexico. The program will consist of lectures by invited speakers in the morning and short papers and discussion periods in the afternoon. Among the topics in hydrodynamics discussed will be stability, magnetohydrodynamics, compressible flow in two dimensions, turbulence, and numerical approaches to hydrodynamics problems. Attendance at the meeting will be by invitation. Information about the meeting can be obtained from Dr. George N. White, Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

TRAVEL GRANTS OF THE NATIONAL SCIENCE FOUNDATION. The National Science Foundation is authorized to provide a limited number of travel grants for the purpose of assisting qualified scientists who wish to attend certain conferences, congresses, and other meetings of scientists held abroad which are concerned with research in the sciences. Decisions will be made by the Foundation concerning specific meetings for which travel support can be extended.

Physical scientists, mathematicians, and engineers who wish support to attend international meetings concerned with subjects in the physical sciences should submit application forms for financial assistance prior to June 30 for travel between October 1 and March 31, and prior to December 31 for travel to be performed between April 1 and September 30. Requests for application forms should be addressed to the Assistant Director for Mathematical, Physical, and Engineering Sciences, National Science Foundation, Washington 25, D. C.

Separate announcements will be made in appropriate journals with reference to specific international meetings dealing with biological and medical subjects for which the Foundation can extend travel aid and the dates when travel applications will be received.

A GRANT TO YALE UNIVERSITY FOR STUDY TO IMPROVE MATHEMATICS INSTRUCTION. The National Science Foundation has announced award of a grant of $100,000 to Yale University to initiate the work of the School Mathematics Study Group to improve instruction in mathematics in United States secondary schools above the sixth grade, including junior high schools.

Under terms of the Foundation grant, it is contemplated that the Study Group will consider preparation of textbook materials, together with teacher's manuals; the training of teachers in the use of such new materials; preparation of monographs designed for better students, secondary-school teachers, and the general educated public; study of various kinds of teaching aids, including films; experimental programs for gifted children; psychological studies of concept formation in mathematics and of attitudes toward mathematics, and the like. The Study Group director will from time to time submit reports to the Foundation, including a general synopsis of the program and the director's evaluation of accomplishments.

THE CAPTAIN ROBERT DEXTER CONRAD AWARD FOR SCIENTIFIC ACHIEVEMENT IN THE NAVY. This award, which was established in 1957 by the Department of the Navy, is the highest scientific award in the Navy. The recipient of the award is selected for his notable contributions to the advancement of naval science. The Navy hopes to invest this annual award for scientific achievement with a prestige that will cause it to rank with such outstanding awards

as the Harmon International Trophy and other equally well-known awards. The 1957 recipient of the award was Dr. Alan T. Waterman, Director of the National Science Foundation. On last March 11, the 1958 award was presented to Dr. Charles C. Lauritsen, Professor of Physics, California Institute of Technology.

A SECOND EDITION (REVISED) OF A. GROTHENDIECK'S "ESPACES VECTORIELS TOPOLOGIQUES" is now available, according to a letter recently received from the Sociedade de Matemática de São Paulo. This revised edition is over 400 pages long and the price is $5.00 postpaid. Copies should be ordered from Professor L. H. Jacy Monteiro at the following address: Sociedade de Matemática de São Paulo, R. Maria Antonia, 258, São Paulo, Brazil, South America.

The Sociedade also announces that the following other titles are available:

(1) J. L. Koszul: "Faisceau et Cohomologie", 58 pp. US $1.00 (postpaid)

(2) J. L. Koszul: "Varietées Kaehleriennes", 100 pp. US $1.50 (postpaid)

PRICING OF THE NOTICES. The transfer of the abstracts of the papers presented at the meetings from the BULLETIN to the NOTICES has made it necessary to establish a price schedule for the NOTICES. Formerly the NOTICES was distributed free of charge to members and to a few other individuals who are actively interested in the Society. The schedule of prices is as follows:

Subscription prices	
List price	$7.00
Agents' price	6.00
Members' price	5.25
Single copies	
Current volume	
List price	$2.00
Agents' price	1.70
Members' price	1.50
Special price at meetings	1.00
Back numbers (none available before 1958)	
Complete volumes, seven issues	
List price	$14.00
Agents' price	11.90
Members' price	10.50

Single issues	
List price	$2.00
Agents' price	1.70
Members' price	1.50

Agents' discount is allowed only for material mailed directly from the Society to foreign addresses.

The attention of the members of the Society is particularly invited to the special provision made for selling the NOTICES at the meetings at a reduced price. It is thought that most persons wishing to buy copies at the meetings will be doing so merely because they have left their own subscription copies at home, and it seems undesirable to request the full price under such circumstances. As most of the readers of the NOTICES know, this journal is published on an extremely tight schedule and it is therefore inevitable from time to time that because of delays in the mails, some members will not receive their NOTICES in time to take to the meetings. A special procedure has been set up to distribute copies free of charge at the meetings to members having such an experience. The procedure is this: Although all copies of the NOTICES distributed at the meetings will be sold at the special price of $1.00, nevertheless a subscriber who buys a copy and claims that his own subscription copy was not delivered in time to take to the meeting will be given a card to fill in and mail to the Headquarters Offices in Providence, which will then return his $1.00.

We take this opportunity to modify a statement which appeared in the November 1957 NOTICES on page 29 in the announcement of the transfer of the abstracts from the BULLETIN to the NOTICES. It is stated there that at the end of the year bound volumes of the NOTICES will be available for purchase. It has now been decided to stock and sell back numbers of the NOTICES (starting with the current volume) in the same way that the back numbers of the other journals of the Society are stocked and sold. This operation does not include the binding of volumes of back numbers. Therefore in years to come back volumes of the NOTICES starting with the 1958 volume may be purchased from the Society, but bound volumes will not be available.

PERSONAL ITEMS

(This section is restricted to members of the Society)

The National Science Foundation has announced the following fellowship awards:

(a) Science Faculty Fellowships: D. T. Finkbeiner, Kenyon College; Howard Levi, Columbia University; Abba V. Newton, Vassar College; Alice T. Schafer, Connecticut College.

(b) Senior Postdoctoral Fellowships: W. G. Bade, University of California, Berkeley; H. W. Kuhn, Bryn Mawr College; H. E. Rauch, New York University; H. L. Royden, Stanford University; I. E. Segal, University of Chicago.

Professor Morris Kline of New York University will be on leave of absence in 1958-59, and has received a Fulbright Grant to lecture at the Technische Hochschule in Aachen, Germany.

Professor M. F. Rosskopf of Teachers College, Columbia University, has received a Fulbright Award to lecture at the Pedagogisk Seminar, University of Oslo, during the year 1958-59.

Dr. A. W. Adler of Princeton University has been appointed a lecturer at the Massachusetts Institute of Technology.

Professor A. A. Albert of the University of Chicago has been appointed to a visiting professorship at the University of California, Los Angeles, for the period March 15, 1958 to June 15, 1958.

Assistant Professor R. E. Allan of Southwestern Louisiana Institute has accepted a position as engineer for Texas Instruments, Inc., Dallas, Texas.

Dr. G. E. Backus of Princeton University has been appointed to an assistant professorship at Massachusetts Institute of Technology.

Mr. T. S. Bright of the Girdler Company has accepted a position as senior mechanical engineer for Burns and Roe, Inc., Melbourne, Florida.

Mr. A. S. Cahn, Jr. of Hartford, Connecticut has accepted a position as mathematician for RAND Corporation, Santa Monica, California.

Dr. R. L. Dunn of Iowa State College has accepted a position as engineering specialist at Hoffman Laboratories, Los Angeles, California.

Mr. E. S. Eby of the University of Illinois has accepted a position as mathematician for the U.S.N. Underwater Sound Laboratory, New London, Connecticut.

Dr. H. P. Flatt of Ramo-Wooldridge Corporation has accepted a position as senior research engineer for Atomics International, Inc., Canoga Park, California.

Associate Professor Abe Gelbart of Syracuse University has been appointed director of the Graduate School of Mathematics, Yeshiva University.

Dr. W. W. Hammerschmidt of the U. S. Air Force has been appointed chief of the Atomic Energy Division, Defense Department, Washington, D. C.

Mrs. Helen W. Henderson of the Rice Institute has accepted a position as mathematician with the Shell Oil Company, Houston, Texas.

Assistant Professor T. C. Holyoke of Miami University has been appointed to an associate professorship at Antioch College.

Dr. M. A. Kervaire of the Institute for Advanced Study has been appointed a research associate at the Battelle Memorial Institute, Carouge, Switzerland.

Mr. P. L. Kingston of the University of Rochester has accepted a position as applied scientist for the International Business Machines Corporation, Syracuse, New York.

Dr. Shoshichi Kobayashi of the Institute for Advanced Study has been appointed a research associate at the Massachusetts Institute of Technology.

Associate Professor Max Kramer of the New Mexico College of Agriculture and Mechanic Arts has been appointed to an assistant professorship at San Jose State College.

Dr. Masatake Kuranishi of Massachusetts Institute of Technology has been appointed to a professorship at Nagoya University, Nagoya, Japan.

Dr. Paolo Lanzano of Douglas Aircraft Company has accepted a position as a member of the technical staff of Ramo-Wooldridge Corporation, Los Angeles, California.

Dr. Joseph Lehner has been appointed to a professorship at Michigan State University.

Dr. Eugene Levin of the Ramo-Wooldridge Corporation has accepted a position as mathematician with the RAND Corporation, Santa Monica, California.

Dr. H. M. Lieberstein of the Ramo-Wooldridge Corporation has accepted a position as design specialist with CONVAIR, San Diego, California.

Mr. E. S. Lowry of the University of Toronto has accepted a position as computing specialist with Avro Aircraft Ltd., Toronto, Ontario, Canada.

Professor L. E. Mehlenbacker of the University of Detroit has been appointed a visiting professor and director of the Summer Institute for High School Teachers of Science and Mathematics at the University of North Dakota.

Dr. J. A. Meier of Michigan State University has accepted a position as research mathematician with the Armstrong Cork Company, Lancaster, Pennsylvania.

Professor Leopoldo Nachbin of the University of Brazil is on leave and has been appointed a Fellow at the Institute for Advanced Study.

Dr. F. P. Peterson of Princeton University has been appointed to an assistant professorship at Massachusetts Institute of Technology.

Dr. M. M. Phelps of Harvard University has been appointed a staff member at Lincoln Laboratory, Massachusetts Institute of Technology.

Mr. H. L. Platzer of Franklin Institute has accepted a position as development engineer for Burroughs Corporation, Paoli, Pennsylvania.

Dr. John Raleigh of the University of Maryland has been appointed to an assistant professorship at Lehigh University.

Assistant Professor D. B. Ray of Massachusetts Institute of Technology is on leave and has been appointed a member of the Institute for Advanced Study.

Associate Professor Abraham Robinson of the University of Toronto has been appointed to a professorship at Hebrew University.

Mr. G. S. Rogers of the State University of Iowa has been appointed to an assistant professorship at the University of Arizona.

Dr. Saul Rosen of the Burroughs Corporation has accepted a position as programming research and development manager of the G and I Division of the Philco Corporation, Philadelphia, Pennsylvania.

Dr. N. E. Sexauer of the University of Illinois has been appointed to an assistant professorship at Ohio University.

Dr. Bella M. Shiffman of the Ramo-Wooldridge Corporation has accepted a position as associate mathematician with the System Development Corporation, Santa Monica, California.

Associate Professor Bethumne Vanderburg of Portland State College has accepted a position as mathematician with the System Development Corporation, Santa Monica, California.

Dr. D. H. Wagner has resigned from the Burroughs Corporation to form the consulting partnership Kettelle and Wagner. For the spring semester, 1958, he filled a position as lecturer at Swarthmore College.

Professor F. M. Weida of the George Washington University has retired with the title Professor Emeritus.

Dr. Sadayuki Yamamuro of the Institute for Advanced Study has been appointed to an assistant professorship at Hokkaido University.

The following promotions are announced:

Allen Devinatz, Washington University, to an associate professorship.

F. W. Dresch, Stanford Research Institute, to manager of the Industrial Operations Research and Electronic Data Processing Division.

Leo Katz, Michigan State University, to a professorship.

E. D. McCarthy, University of Detroit, to a professorship.

J. C. Moore, Princeton University, to an associate professorship.

Pasquale Porcelli, Illinois Institute of Technology, to an associate professorship.

O. M. Rasmussen, University of Denver, to an associate professorship.

J. K. Reckzeh, State Teachers College, Jersey City, New Jersey, to a professorship.

B. V. Ritchie, University of Detroit, to an associate professorship.

J. E. Rowe, University of Michigan, to an associate professorship.

P. W. Shaw, San Diego State College, to an assistant professorship.

W. L. Shepherd, Texas Western College, to an assistant professorship.

R. J. Silverman, Illinois Institute of Technology, to an associate professorship.

I. M. Singer, Massachusetts Institute of Technology, to an associate professorship.

A. J. Tingley, Dalhousie University, to an associate professorship.

R. L. Vitale, New York City Transit Authority, to electrical engineer.

L. R. Wilcox, Illinois Institute of Technology, to a professorship.

The following appointments to instructorships are announced:

University of Alabama: Mr. H. C. Miller, Jr.; Brooklyn College: Dr. L. L. Gavurin; Harvey Mudd College: Mr. R. T. Seeley; Massachusetts Institute of Technology: Mr. E. R. Rodemich, Mr. J. W. Smith, Dr. J. J. Yeh; University of Oklahoma: Mr. W. J. Cody, Jr.; Southern Union College: Mr. H. H. Diehl.

Deaths:

Dr. Walter Bartky, Vice President of the University of Chicago, died on March 19, 1958, at the age of fifty-six years. He had been a member of the Society for thirty-one years.

Miss Phyllis H. Belisle of Agnes Irwin School died January 24, 1958, at the age of thirty years.

Mr. Ray S. Hoyt of Chatham, New Jersey, died on March 16, 1958, at the age of seventy-five years. He had been a member of the Society for forty years.

LETTERS TO THE EDITOR

Editor, the NOTICES.

Recently, an author submitted identical manuscripts simultaneously to the editors of at least three journals, apparently intending to withdraw them from the less prompt ones. Since the subject of mathematical ethics is missing from the standard curriculum, and has not yet been covered by the otherwise comprehensive Bourbaki, a few comments are in order. The author in question apparently did not realize that the n-tuple submission of a manuscript makes n-tuple demands on the time of editors and referees. These people do not edit or referee for fun, but out of a sense of duty, and it is unfair to make them work harder. Also, n-tuple submission of papers involves unnecessary expense in postage and secretarial time, when $n > 1$, and to this extent it decreases the amount of mathematics that can be published. The possible gain for the author is in any case small, since most manuscripts are handled promptly, even though everybody knows of a few glaring counter-examples.

It would perhaps help authors avoid temptation if all editors of mathematical journals would adopt the principle that only typescripts are acceptable: no carbon copies, no mimeographed reports, no purple ink. This is desirable for other reasons as well; and here we add the following reminder: All typescripts must be double- or triple-spaced (including the list of references and the footnotes) on reasonably good, opaque paper, and with reasonable margins. The references should be accurate and complete; proper abbreviations for names of journals may usually be obtained from MATHEMATICAL REVIEWS; in case of doubt, consult a recent issue of the journal to which the paper is to be submitted. Titles of references and authors' names should be spelled correctly, even when they are not in English, with particular attention to accents. Carelessly prepared papers are in danger of receiving a low priority rating, not only from harassed referees, but also from ill-tempered editors.

R. P. Boas
Northwestern University
G. Piranian
University of Michigan

NOTE:

The purpose of this new department is to provide a forum for discussions of the programs of the Society, and a method for communicating information of interest to the membership. Questions concerning matters of scholarship, such as those relating to the location of primary references, will be welcomed.

The Council has instructed the Editor of the NOTICES not to allow the new department to be used for quick publication of mathematical results, and not to accept criticism of specific individual papers or of specific individual reviews in MATHEMATICAL REVIEWS or elsewhere. Thus a letter protesting against what the writer feels to be an unfair review in MATHEMATICAL REVIEWS of his or someone else's results is not eligible. There will be no editing of any letter, except for correction of obvious minor misprints, without direct negotiations with the writer.

The deadline for each issue of the NOTICES for all letters and news items is the same as the deadline for abstracts, as announced on the inside front cover page of earlier issues.

MEMORANDA TO MEMBERS

Note: It has frequently been suggested that it would be desirable to print a list of the current officers and committees of the Society once a year in the NOTICES. We shall try to do that in the June issue each year.

A question might arise as to whether this could be done earlier in the calendar year - at least in the April NOTICES. The answer is in the affirmative in the case of the principal officers, but the information would be defective in the case of the committees. Various questions arise at the beginning of each calendar year in connection with chairmanships and other committee matters which require two or three months to settle, and the deadline for the April NOTICES is usually early in March. Here is the 1958 list:

AMERICAN MATHEMATICAL SOCIETY
DIRECTORY OF OFFICERS AND COMMITTEES

1958

OFFICERS

		Term Expires December 31
President	Richard Brauer	1958
President Elect	E. J. McShane	1958
Vice Presidents	Garrett Birkhoff	1959
	Salomon Bochner	1958
	N. E. Steenrod	1959
Secretary	J. W. Green	1958
Associate Secretaries	G. B. Huff	1959
	V. L. Klee, Jr.*	1959
	R. D. Schafer	1958
	J. W. T. Youngs	1959
Treasurer	A. E. Meder, Jr.	1958

BOARD OF TRUSTEES

Officers of the Society, ex officio

A. E. Meder, Jr., Secretary of the Board	1958
Richard Brauer, Presiding Officer	1958

Elected Trustees

H. F. Bohnenblust	1962
G. A. Hedlund	1958
W. T. Martin	1960
Deane Montgomery	1961
Mina S. Rees	1959

* W. S. Pierce substituting for V. L. Klee, Jr. from June 1, 1958 to August 31, 1959.

Term Expires
December 31

PUBLICATIONS COMMITTEES

BULLETIN Editorial Committee
- E. E. Moise 1960
- J. C. Oxtoby, Chairman 1958
- B. J. Pettis 1959

PROCEEDINGS Editorial Committee
- R. P. Boas, Jr. 1958
- P. R. Halmos 1960
- Irving Kaplansky, Chairman 1959
- Hans Samelson 1958

Associate Editors
- Raoul Bott 1958
- Paul Civin 1958
- Edwin Hewitt 1958
- R. V. Kadison 1959
- Erwin Kleinfeld 1960
- Jacob Korevaar 1959
- M. O. Reade 1960
- I. M. Singer 1959
- R. M. Thrall 1958
- John Wermer 1959

COLLOQUIUM Editorial Committee
- Salomon Bochner, Chairman 1960
- Deane Montgomery 1958
- E. J. McShane 1959

MATHEMATICAL SURVEYS Editorial Committee
- Irving Kaplansky 1960
- I. J. Shoenberg, Chairman 1958
- S. M. Ulam 1959

MATHEMATICAL REVIEWS Editorial Committee
- R. P. Boas, Jr., Chairman 1959
- J. L. Doob 1960
- W. S. Massey 1958

TRANSACTIONS and MEMOIRS Editorial Committee
- S. S. Chern* 1959
- Samuel Eilenberg, Chairman 1960
- G. P. Hochschild 1960
- Mark Kac 1958

* Editor for Memoirs

Term Expires
December 31

Associate Editors

Lipman Bers 1961
E. A. Coddington 1962
J. A. Dieudonné 1962
Harley Flanders 1962
A. M. Gleason 1958
L. A. Henkin 1961
G. A. Hunt 1961
G. P. Hochschild 1960
R. V. Kadison 1962
Charles Loewner 1959
M. M. Schiffer 1961
I. E. Segal 1959
H. C. Wang 1961

AMERICAN JOURNAL OF MATHEMATICS, Representatives of the Society on the Board of Editors

A. M. Gleason 1959
J. A. Dieudonné 1958

Committee on Printing and Publishing

E. F. Beckenbach 1960
E. G. Begle, Chairman 1959
A. W. Tucker 1958

COUNCIL

Members at large

R. E. Bellman 1959
Lipman Bers 1959
R. H. Bing 1960
A. M. Gleason 1959
P. R. Halmos 1959
Einar Hille 1958
J. L. Kelley 1958
D. H. Lehmer 1959
L. H. Loomis 1960
P. C. Rosenbloom 1960
Walter Rudin 1960
Hans Samelson 1958
I. E. Segal 1960
A. H. Taub 1958
A. D. Wallace 1958

Term Expires
December 31

Ex officio Members

Officers

Garrett Birkhoff 1959
Salomon Bochner 1958
Richard Brauer 1958
J. W. Green 1958
G. B. Huff 1959
V. L. Klee, Jr. 1959
E. J. McShane 1958
A. E. Meder, Jr. 1958
R. D. Schafer 1958
N. E. Steenrod 1959
J. W. T. Youngs 1959

Ex Secretary

E. G. Begle 1958

Members of Publications Committees

E. F. Beckenbach 1960
E. G. Begle 1959
R. P. Boas, Jr. 1959
Salomon Bochner 1960
S. S. Chern 1959
J. A. Dieudonné 1958
J. L. Doob 1960
Samuel Eilenberg 1960
A. M. Gleason 1959
P. R. Halmos 1960
G. P. Hochschild 1960
Mark Kac. 1958
Irving Kaplansky 1960
W. S. Massey 1958
E. J. McShane 1959
E. E. Moise 1960
Deane Montgomery 1958
J. C. Oxtoby 1958
B. J. Pettis 1959
Hans Samelson 1958
I. J. Schoenberg 1958
A. W. Tucker 1958
S. M. Ulam 1959

EXECUTIVE COMMITTEE

R. H. Bing 1959
Einar Hille 1959
W. S. Massey 1958
A. H. Taub 1958

Term Expires
December 31

Ex officio Members

J. W. Green 1958
Richard Brauer 1958
E. J. McShane 1958

STANDING COMMITTEES

Committee to Select Hour Speakers for Summer and Annual Meetings

J. W. Green, Chairman 1958
Einar Hille 1958
R. L. Wilder 1959

Committee to Select Hour Speakers for Eastern Sectional Meetings

Warren Ambrose 1958
R. D. Schafer, Chairman 1958
D. C. Spencer 1959

Committee to Select Hour Speakers for Southeastern Sectional Meetings

O. G. Harrold 1958
G. B. Huff, Chairman 1959
A. D. Wallace 1959

Committee to Select Hour Speakers for Western Sectional Meetings

Raoul Bott 1959
R. C. Buck 1958
J. W. T. Youngs, Chairman 1959

Committee to Select Hour Speakers for Far Western Sectional Meetings

H. F. Bohnenblust 1959
V. L. Klee, Jr., Chairman 1959
Charles Loewner 1958

Committee on Applied Mathematics

R. E. Bellman 1959
Garrett Birkhoff, Chairman 1958
R. S. Burington 1959
Shizuo Kakutani 1958
Brockway McMillan 1960
S. M. Ulam 1960

Term Expires
June 30

Committee on Translations from Russian and other Foreign Languages

R. E. Bellman 1960
R. P. Boas, Jr. 1958
Edwin Hewitt* 1961
Irving Kaplansky 1958
E. R. Kolchin* 1961
Hans Samelson 1960
J. V. Wehausen, Chairman 1959

*Term starts July, 1958

Organizing Committee for Summer Institutes

R. H. Bing, Chairman 1959
H. S. M. Coxeter 1959
Nelson Dunford 1958
A. M. Gleason 1960
E. R. Kolchin 1958
R. S. Phillips 1960

AD HOC COMMITTEES

Committee of Trustees to Handle Investment Problems for the Society

W. T. Martin
A. E. Meder, Jr., Chairman
Mina S. Rees

Committee to Recommend to the Council Desirable Extensions of the Society's Activities

A. A. Albert
Wilfred Kaplan
J. L. Kelley, Chairman
W. T. Martin
William Prager
A. W. Tucker

Committee on Mathematicians Dismissed for Political Reasons

W. L. Duren, Jr.
R. D. James
H. W. Kuhn
E. E. Moise, Chairman
J. B. Rosser

Committee on Corporate Memberships

M. M. Flood
B. P. Gill
C. C. Hurd, Chairman

Committee to Investigate Present Economic Status of Teachers

A. A. Albert
R. E. Bellman
David Blackwell
G. N. Garrison
Wallace Givens, Chairman
J. W. Green
H. M. Schaerf

Committee to Consider the Establishment of Annual Examinations at the Master's Level

R. P. Dilworth — Leonard Gillman, Chairman
D. E. Richmond

Committee to Recommend Gibbs Lecturers for 1958 and 1959

Lipman Bers — Wilfred Kaplan
S. M. Ulam, Chairman

Committee to Award the Bôcher Prize in 1958

Salomon Bochner, Chairman — J. L. Doob
D. V. Widder

Program Committee for a Symposium on Combinatorial Design and Analysis

Richard Bellman — S. A. Schelkunoff
Marshall Hall, Chairman — Claude Shannon
C. B. Tompkins

Committee to Review the Arrangements Between the Society and the Johns Hopkins Press

Harish-Chandra — A. E. Meder, Jr.
A. W. Tucker

Invitations Committee for Summer Institute, 1958

Lamberto Cesari — Tibor Rado, Chairman
Herbert Federer — L. C. Young
J. W. T. Youngs

Invitations Committee for Summer Institute, 1959

P. T. Bateman — D. H. Lehmer
A. T. Brauer — Ivan Niven
B. W. Jones, Chairman — A. L. Whiteman

Committee to Nominate Officers and Members of the Council for 1959

Herbert Busemann — George Mackey, Chairman
Marshall Hall — B. J. Pettis
G. de B. Robinson

Committee on Expository Books

Lipman Bers — A. M. Gleason
Salomon Bochner — E. J. McShane, Chairman
Deane Montgomery

JOINT COMMITTEES

Committee of the Society and Association on Places of Meetings

G. R. MacLane 1960
R. D. Schafer 1959
R. M. Thrall, Chairman 1958

Committee on Arrangements for the Summer Meetings at Cambridge, Massachusetts, August 1958

J. H. Curtiss
Philip Franklin
H. M. Gehman
F. B. Hildebrand
Norman Levinson
Hartley Rogers, Jr.
R. D. Schafer
G. B. Thomas, Chairman

Committee on Arrangements for the Annual Meetings at Philadelphia, Pennsylvania, January 1959

P. A. Caris
J. H. Curtiss
Robert Ellis
H. M. Gehman
W. H. Gottschalk
Emil Grosswald, Chairman
R. D. Schafer
G. E. Schweigert
C. T. Yang

Committee on Employment Opportunities

W. M. Hirsch, Chairman
R. F. Rinehart
A. E. Taylor

CONFERENCE ORGANIZATION OF THE MATHEMATICAL SCIENCES

Representatives of the American Mathematical Society

Richard Brauer
J. W. Green
E. J. McShane, Alternate

Representatives of the Mathematical Association of America

H. M. Gehman
G. B. Price

Representatives of the Association for Symbolic Logic

H. B. Curry
P. R. Halmos

Representatives of the Institute of Mathematical Statistics

J. F. Daly
H. B. Mann

Representatives of the National Council of Teachers of Mathematics

H. F. Fehr
J. R. Mayor

Representatives of the Society for Industrial and Applied Mathematics

D. B. Houghton
T. H. Southard

Members-at-large

I. E. Block
H. W. Kuhn
A. E. Meder, Jr.
Marie S. Wilcox
R. L. Wilder
S. S. Wilks

APPOINTED REPRESENTATIVES OF THE SOCIETY

	Term Expires December 31
To the U. S. National Committee on Theoretical and Applied Mechanics	
William Prager	1960

Term Expires
December 31

On the Council of the American Association for the Advancement of Science

W. L. Duren 1959
G. A. Hedlund 1959

On the Editorial Board of the Duke Mathematical Journal

R. H. Fox 1958
I. E. Segal 1958

On the Board of Editors of the Annals of Mathematics

A. M. Gleason 1960
J. L. Kelley 1960
L. V. Ahlfors 1960

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FEDERAL SCHOOL OF TECHNOLOGY

The following item may be ordered from: Federal School of Technology, Zurich, Switzerland.

A. Borel, "Cohomologie des espaces localement compacts, d'après J. Leray". 2nd edition, 1957. Sw. fr. 7.00

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Austern, N., and Percus, J. K. Propagation of strong blast in an atmosphere of varying density. AEC Computing and Applied Mathematics Center, Institute of Mathematical Sciences, New York University, 1957. 35 pp.

Barenblatt, G. I. On self-similar solutions of the Cauchy problem for the nonlinear parabolic equation of the nonsteady filtration of a gas in a porous medium. Trans. by M. D. Friedman. Needham Heights, 1957. 6 pp.

Belotserkovskii, O. M. Flow past a circular cylinder with a detached shock. Trans. by M. D. Friedman. Needham Heights, 1957. 7 pp.

Blokh, E. L., and Kharkevich, A. A. Geometric theory of the threshold of the capacity of a communication system. Trans. by M. D. Friedman. Needham Heights, 1957. 6 pp.

Bremmer, H. The mode expansion in the low-frequency range for propagation through a curved stratified atmosphere. (National Bureau of Standards, Rep. 5518.) Boulder, National Bureau of Standards, 1957. 21 pp.

Cheney, W. See Goldstein, A. A.

Davenport, W. B. Jr., and Root, W. L. An introduction to the theory of random signals and noise. New York, McGraw-Hill, 1958. 9 + 393 pp. $10.00

Fok, V. A. Theory of diffraction from a paraboloid of revolution. Trans. by M. D. Friedman. Needham Heights, 1957. 41 pp.

Forbes, G. F. Digital differential analyzers. 4th ed. Pacoima, 1957. 201 pp.

Friedman, M. D. See Barenblatt, G. I.; Belotserkovskii, O. M.; Blokh, E. L. ; Fok, V. A.; Gandin, L. S.; Ivlev, D. D.; Kolmogorov, A. N.; Lunts, A. G.; Nikol'skii, V. V.: Povarov, G. N.; Roginskii, V. N.; Tsetlin, M. L.; Tsurkov, I. S.; Ulanov, G. M.; Zhuravlev, Iu. I.

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Ivlev, D. D. Buckling of eccentric pipes. Trans. by M. D. Friedman. Needham Heights, 1957. 7 pp.

Jordan, C. Traité des substitutions et des équations algébriques. New printing. Paris, Blanchard, 1957. 18 + 667 pp. 7500 fr.

Kharkevich, A. A. See Blokh, E. L.

Kolmogorov, A. N. Statistical theory of oscillations with a continuous spectrum. Trans. by M. D. Friedman. Needham Heights, 1957. 12 pp.

Liniger, W. Zur Stabilität der numerischen Integrationsmethoden für Differentialgleichungen. Université de Lausanne, 1957. 95 pp.

Lunts, A. G. Application of Boolean matrix algebra to the analysis and synthesis of relay-switching circuits. Trans. by M. D. Friedman. Needham Heights, 1957. 5 pp.

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Percus, J. K. See Austern, N.

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Roginskii, V. N. Equivalent transformations of class II relay circuits. Trans. by M. D. Friedman. Needham Heights, 1957. 7 pp.

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Soloveichik, R. E. See Gandin, L. S.

Tsetlin, M. L. Use of matrix calculus to synthesize relay-switching circuits. Trans. by M. D. Friedman. Needham Heights, 1957. 7 pp.

Tsurkov, I. S. On elasto-plastic equilibrium of shells of revolution for small axisymmetric deformations. Trans. by M. D. Friedman. Needham Heights, 1957. 7 pp.

Ulanov, G. M. Invariance to ϵ in linear combination automatic regulation systems. Trans. by M. D. Friedman. Needham Heights, 1957. 6 pp.

Wait, J. R. The propagation of V. L. F. pulses to great distances. (National Bureau of Standards, Rep. 5513.) Boulder, National Bureau of Standards, 1957. 56 pp.

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Zhuravlev, Iu. I. On the separability of subsets of the vertices of an n-dimensional unit cube. Trans. by M. D. Friedman. Needham Heights, 1957. 6 pp.

ABSTRACTS OF CONTRIBUTED PAPERS

THE ANNUAL MEETING IN CINCINNATI, OHIO

January 28-30, 1958

542-126. F. A. Raymond: A new proof for duality theorems in generalized manifolds.

Let X, locally compact, be a countable union of compact sets. Let $\mathcal{K}$ denote the associated sheaf to the complex K defined in my paper Cech homology from a chain complex. Let $\Gamma_c(\mathcal{K})$ be the sections with compact support. The injection $K_c \to \Gamma_c(\mathcal{K})$ is an isomorphism onto. Suppose, in addition, X is an orientiable generalized n-manifold in the sense of Wilder. One can show that the general duality theorem for carapaces of H. Cartan applies here and we obtain an isomorphism of the Cech homology sequence (compact carriers) $H_p^c(U) \to^{i^*} H_p^c(X) \to^{j^*} H_p^c(X,U) \to^{\partial^*}$ onto the compact Cech cohomology sequence $H_c^{n-p}(U) \to^{i^*} H_c^{n-p}(X) \to H_c^{n-p}(X-U) \to^{\delta^*}$ such that the vertical diagram is commutative. (U is any open set in X). We actually need only to assume X is finite dimensional, and r-cole may be replaced by a weaker assumption. One can obtain (a) duality theorems for locally orientiable manifolds, (b) duality theorems in terms of "infinite homology" and ordinary Cech cohomology, (c) duality theorems when the coefficient group is not a field (although the homology will not necessarily be Cech homology). I have been informed that A. Borel has also obtained a new proof of the duality theorems for generalized manifolds. (Received December 12, 1957.)

542-127. F. A. Raymond: On factorization of generalized manifolds.

Since we are able to prove the Poincaré and Alexander-Pontryagin types of duality with only a finite dimensional instead of an explicit n-dimensional assumption we shall only require our manifolds to be finite dimensional. Nothing significant is lost from this approach, [separable metric locally orientiable 2-manifolds are still locally Euclidean]. Let M be a paracompact product space $M = A \times B$. A necessary and sufficient condition that M be an orientiable (locally orientiable) generalized manifold is that both A and B be orientiable (locally orientiable) generalized manifolds. This generalizes results of K. Borsuk (Fund. Math. vol. 33 (1945) pp. 273-298) and G. S. Young (Proc. Amer. Math. Soc. vol. 1 (1950) pp. 215-223). It follows, in the orientiable case, that codimen-

sion A + codimension B = codimension M = n. The work for these three papers has been sponsored in part by A. F. - 49 (638) - 104. (Received December 12, 1957.)

542-128. Heinz Renggli: An inequality for logarithmic capacities.

Let E denote the complex z-plane, and let A, A $\subset$ E, and B, B $\subset$ E, be compact. By C(A) resp. C(B) we denote the logarithmic capacity of A resp. B [R. Nevanlinna, Eind. analyt. Funkt., Springer, 1936]. Then we have proved the following inequality $C(A \cup B)\cdot C(A \cap B) \leqq C(A)\cdot C(B)$ [cf. G. Choquet (review of B. Lepson) Math. Rev. vol. 18 (1957) p. 295]. This inequality is proved with the aid of a similar inequality stated for extremal lengths [L. Ahlfors and A. Beurling, Acta Math. vol. 83 (1950) p. 101]: if $\{\gamma\}$ and $\{\gamma'\}$ are two families of curves which satisfy some additional conditions and if $\lambda[\{\gamma\}]$ denotes the extremal length of a family $\{\gamma\}$, then we have $1/\lambda[\{\gamma\} \cup \{\gamma'\}] + 1/\lambda[\{\gamma\} \cap \{\gamma'\}] \leqq 1/\lambda[\{\gamma\}] + 1/\lambda[\{\gamma'\}]$. Furthermore we need the relation between reduced extremal length and logarithmic capacity [V. Wolontis, Amer. J. Math. vol. 74 (1952) p. 587]. (Received December 12, 1957.)

542-129. Andrew Sobczyk: Discrete random processes. Preliminary report.

The closed interval $0 \leqq \omega \leqq 1$ is regarded as index set for a random process $\Omega = \{x_\omega(t)\}$ of sample functions $x_\omega(t)$ on the set of real integers $T = \{\ldots -2, -1, 0, 1, 2, \ldots\}$. A particular class of processes Ω then consists of those for which the probability measure is ordinary Lebesgue measure on [0,1]. A process is pseudo-ergodic if Ω is a subset of a hyperplane $A_0 = A(x) = \lim_{k\to\infty}(1/2k + 1)\sum_{t=-k}^{t=k} x(t)$ in the Banach space (m) of all bounded functions x(t) on T (see a previous abstract by the author). In case the functions $x_\omega(t)$ converge to constants $x_\omega(+\infty)$ and $x_\omega(-\infty)$ as $t \to +\infty$ and as $t \to -\infty$, the process is pseudo-ergodic. Implications for random processes, of the fact that there exists no continuous projection of (m) on the subspace (c) of convergent sequences x(t), are explored. (Received December 12, 1957.)

542-130. S. R. Struik: Bolzano and the historic significance of his contributions to modern calculus.

The publication of Bolzano's work by the government of Czechoslovakia, including the sifted, voluminous mathematical manuscripts, dormant in Vienna's Archivs since his death in 1848, affords an evaluation of this predecessor of

Cauchy and Weierstrass. Bolzano, a devout catholic priest, is the first to have laid the exact foundation of infinitesimal calculus. Thus Prague, at the crossroads of Europe, ideologically and physically, is the cradle of 19th century fundamentally new thoughts that in the perfected form of modern calculus are the prerequisite of 20th century physics, culminating in the experimental proof of Newton's laws by adding a rotating heavenly body of our own making to those existing. Similarly, and not just by chance, Prague was the cradle of revolutionary protestant thought in Hus, the predecessor of Luther and Calvin in the 16th century, that led, with the help of the newly invented firearms, to the devastating war of the 17th century. So revolutionary 19th century infinitesimal calculus (together with new social ideas) leads to 20th century experimental physics and probably to the most devastating war yet. (Received December 12, 1957.)

542-131. H. C. Wang: Invariant connections over a principal fibre bundle.

Let (E,S) be a differentiable principal fibre bundle with total space E and structure group S, and G a Lie group of automorphisms of (E,S). Choose a fibre F of E, and denote by J the subgroup of G leaving F invariant. There is a natural homomorphism ϕ: J $\to$ S. If G is transitive on the fibres of E, the following are proved: I. There is a 1-1 correspondence between the G-invariant infinitesimal connections Γ over (E,S) and linear maps Ψ: $\mathfrak{g} \to \mathfrak{s}$ such that $\Psi \circ \mathrm{Ad}\, j = \mathrm{Ad}\,\phi(j) \circ \Psi$, $j \in$ J, and $\dot{\phi}(\bar{j}) = \Psi(\bar{j})$, $\bar{j} \in \mathfrak{J}$. Here $\mathfrak{g}, \mathfrak{s}, \mathfrak{J}$, denote, respectively, the Lie algebras of G, S, J, and $\dot{\phi}$: $\mathfrak{J} \to \mathfrak{s}$ is the homomorphism induced by ϕ. II. Let $\mathfrak{H}$ denote the linear subspace of $\mathfrak{s}$ spanned by vectors of the form $\Psi[\bar{g}_1, \bar{g}_2] - [\Psi\bar{g}_1, \Psi\bar{g}_2]$, $\bar{g}_1, \bar{g}_2 \in \mathfrak{g}$. Then the Lie algebra of the holonomy group of the connection Γ is given by $\mathfrak{H} + [\Psi(\mathfrak{g}), \mathfrak{H}] + [\Psi(\mathfrak{g}), [\Psi(\mathfrak{g}), \mathfrak{H}]] + \ldots$. (Received December 12. 1957.)

542-132. Chien Wenjen: A characterization of Hewitt's Q-spaces. Preliminary report.

A completely regular space X is said to be a Q-space if every free maximal ideal in C(X,R) is hyper-real. (Hewitt, Trans. Amer. Math. Soc. vol. 64 (1948) p. 85) (1) A completely regular space X is a Q-space iff there exists a compact Hausdorff space B $\supset$ X such that X is the intersection of the members of a family of F_σ's in B. (cf. J. Kelley, General Topology, p. 208) (2) The non-void intersection of a family of Q-spaces contained in a completely regular

space is a Q-space. (3) A subset of a Q-space X is a Q-space iff it is the intersection of a closed subset of X and the members of a family of F_σ's in X. (4) The intersection of all E_σ's containing a completely regular space X and contained in βX is νX. (For definition of νX, see Hewitt, loc. cit. p. 89; cf. Gillman, Henrikson, and Jerison, Proc. Amer. Math. Soc. vol. 5 (1954) Theorem 5. (5) If a completely regular space X is not pseudo-compact, there exist at least $2^{\aleph_0}$ hyper-real maximal ideals in C(X,R). (Received December 12, 1957.)

542-133. WITHDRAWN.

542-134. R. F. Williams: Lebesgue area for compact two-dimensional surfaces.

By a surface is meant a pair (f,X), where X is a compact 2-dimensional (cover definitions) Hausdorff space and f is a map of X into a Euclidian space, E^n. The purpose of this paper is to define Lebesgue area for such surfaces. The role of elementary surface is replaced by 'approximately elementary' surface, defined in terms of open covers α of X and barycentric α-maps. It is shown that the definition given agrees with the usual if X is a compact 2-manifold, (with or without boundary). (Received December 12, 1957.)

542-135. G. S. Young: A generalization of Bagemihl's theorem on ambiguous points.

Let D be a domain in Euclidean n-space, and let f be a function from D into a topological space S. An r-cell I in Bdry D is an r-cell of disjoint cluster sets for f provided there exist two closed (r + 1)-cells J_1 and J_2 each containing I and lying in D except for I such that the cluster set on I from J_1 of f does not meet the cluster set on I from J_2 on f; approximately, the images of the parts of J_1 and J_2 near I are disjoint. It is proved that if S is compact metric and D is the interior of the (n - 1)-sphere then there do not exist uncountably many disjoint cluster sets of f. Bagemihl proved (Proc. Nat. Acad. Sci. U. S. A. vol. 41 (1955) pp. 379-382) essentially the case n - 2. This paper will appear in the Michigan Mathematics Journal. (Received December 12, 1957.)

542-136. G. R. MacLane: On the growth of entire functions.

Let $\phi(t) \in C^2$, $\phi(t) > 0$, $\phi'(t) > 0$, for $t \geqq 0$. Let $f(z) = \sum a_n z^n$ be entire and let M(r) be the maximum modulus function of f. We define λ = the order

of f relative to ϕ by the conditions: $M(r) < \phi(r^{\lambda+\eta})$ for $\eta > 0$ and $r > r_0(\eta)$; $M(r) > \phi(r^{\lambda-\eta})$ for various arbitrarily large r. Let $\Psi(u) = \log \phi(e^u)$. If ϕ satisfies the additional conditions, $\Psi'(u) \uparrow \infty$ and $\liminf(\Psi''/\Psi') > 0$, then $1/\lambda = \liminf(-\log a_n /(nh(n)))$, where h is the function inverse to Ψ'. In the classical case where $\phi(t) = e^t$, h(n) is of course log n. The general result follows from consideration of the con-or-di-vergence of $\int\int |f(re^{it})|^2 \phi^{-2}(r^\mu) r dr dt$ and some tedious asympototic estimates. (Received December 12, 1957.)

542-137. R. E. Chamberlin: A class of quasi-complexes.

A tree-like continuum is given that is imbeddable in the plane but which is not a quasi-complex. The tree-like continua which are quasi-complexes are characterized. The definitions of concepts used in this characterization are rather lengthy and will not be reproduced here. (Received December 12, 1957.)

542-138. Eldon Dyer: On the dimension of products.

Let X and Y denote compact Hausdorff spaces whose covering dimensions, dim X and dim Y, are finite. For the group G let D(X;G) denote the cohomological dimension of X with coefficients in G. For the prime p let R_p denote the additive group of those rationals which in lowest form do not contain p in the denominator. Bockstein has shown that there is a prime p such that dim X = D(X;R_p). In this note it is shown that if dim X × Y = dim X + dim Y, then there is a prime p such that dim X = D(X;R_p) and dim Y = D(Y;R_p). Also, if X and Y are clc and there is a prime p such that dim X = D(X;R_p) and dim Y = D(Y;R_p), then dim X × Y = dim X + dim Y. (The statement that X is clc means that for each point $x \in X$ and closed neighborhood U of x, there is a closed neighborhood V of x, $V \subset U$, such that the inclusion homomorphism $\tilde{H}^*(U;Z) \longrightarrow \tilde{H}^*(V;Z)$ is trivial, Z denoting the ring of integers.) An example of Boltyanskii of a compact metric space B such that dim B × B = 3 shows that the condition of being clc is not redundant. (Received December 12, 1957.)

542-139. Eldon Dyer: Duality theorems for generalized manifolds.

Let X denote a finite dimensional, locally compact, paracompact space. For a field $\mathcal{F}$, X is of type $(m;\mathcal{F})$ provided for each point $x \in X$ the i-th local cohomology group of X at x, $B^i(X,x;\mathcal{F})$, is 0 for $i \neq m$ and is isomorphic to $\mathcal{F}$ for i = m. (For definitions of local cohomology groups and stability see the abstract in this section entitled The local Künneth sequence.) If, in addition,

$B^*(X,x;\mathcal{F})$ is stable for each $x \in X$, X is said to be an n-gm. If X is of type $(n;\mathcal{F})$, it is said to be orientable if it irreducibly carries (with respect to closed subsets) an n-dimensional cohomology class with compact support. Let $H^*_\Phi(\ ;\mathcal{F})$ denote cohomology with closed supports and $H^*_c(\ ;\mathcal{F})$ denote cohomology with compact supports. Then if X is of type $(n;\mathcal{F})$, $H^i_\Phi(X;\mathscr{S}) \cong \mathrm{Hom}(H^{n-i}_c(X;\mathcal{F});\mathcal{F})$, where $\mathscr{S}$ is a sheaf each of whose stalks is isomorphic to $\mathcal{F}$. If either X is locally orientable and $\mathcal{F} = Z_2$ or X is orientable, then $\mathscr{S}$ is the simple sheaf $\mathcal{F}$. If X is a connected locally orientable n-gm and A is a compact proper subset of X, then $H^n(A;\mathcal{F}) = 0$. Finally, if X is a compact, orientable, sphere-like n-gm and A is a closed subset of X such that X-A is paracompact, then $\tilde{H}^i_\Phi(X-A;\mathcal{F}) \cong \mathrm{Hom}(\tilde{H}^{n-i-1}(A;\mathcal{F});\mathcal{F})$. Related results have recently been obtained independently by A. Borel. (Received December 12, 1957.)

542-140. Eldon Dyer: The local Künneth sequence.

The inverse family $(G\alpha,\rho^\alpha_\beta)$, $\alpha \in A$, of groups and homomorphisms is stable provided there exist $\alpha,\beta \in A$, $\beta \succ \alpha$, such that if $\gamma \in A$, $\gamma \succ \alpha$, there is a $\delta \in A$, $\delta \succ \gamma$ and $\delta \succ \beta$, such that ρ^γ_α maps $\mathrm{im}\rho^\delta_\gamma$ isomorphically onto $\mathrm{im}\rho^\beta_\alpha$. For the locally compact space X and point $x \in X$, the local cohomology ring $B^*(X,x;G)$ with coefficients in G is the inverse limit of the family $(H^*_c(X,X-U;G), \rho^U_V)$, indexed by the family of open neighborhoods of x, where the ρ^U_V are induced by inclusion. The local cohomology ring is stable if this inverse family is stable. The proper onto mapping $f:X \to Y$ is regular at $y \in Y$ provided it is open at y and for each $x \in F_y$, $F_y = f^{-1}(y)$, and closed neighborhood U of x, there exist closed neighborhoods V of x and W of y, $V \subset U$, such that for each $z \in W$, $\tilde{H}^*(U \cap F_z) \to \tilde{H}^*(V \cap F_z)$ is trivial. THEOREM. If X is a finite dimensional compact Hausdorff space, $f:X \to Y$ is a proper onto mapping which is regular at $y \in Y$, $x \in F_y$, and both $B^*(Y,y)$ and $B^*(F_y,x)$ are stable (coefficients the ring of integers), then $B^*(X,x)$ is stable and the sequence $0 \to \sum_{i+j=n} B^i(Y,y) \otimes B^j(F_y,x) \to B^n(X,x) \to \sum_{i+j=n+1} \mathrm{Tor}(B^i(Y,y),B^j(F_y,x)) \to 0$ is exact. If $\mathcal{F}$ is a field, then $\sum_{i+j=n} B^i(Y,y;\mathcal{F}) \otimes B^j(F_y,x;\mathcal{F}) \cong B^n(X,x;\mathcal{F})$. (Received December 12, 1957.)

542-141. P. E. Conner and Eldon Dyer: Singular fiberings by spheres.

A singular fibering (as defined by Montgomery and Samelson) is a quadruple $[(X,A),(Y,B),\pi,F]$ such that (1) $\pi:(X,A) \to (Y,B)$ is a proper open onto map-

ping, (2) $\pi^{-1}(B) = A$ and $\pi|A$ is a homeomorphism, and (3) $\pi|X-A: X-A \to Y-B$ is a fiber mapping with fiber F. The purpose of this note is to show that certain cohomological properties (always with coefficients in Z_2) of X are inherited by A when F is a cohomology r-sphere, $r > 0$. Assume X is finite dimensional and compact Hausdorff. Then (1) if X is cohomologically trivial, so is A. (2) If X is clc, then A is clc (A space Y is clc if for each point $y \in Y$ and closed neighborhood U of y, there is a closed neighborhood V of y, $V \subset U$, such that $\tilde{H}^*(U;Z_2) \to H^*(V;Z_2)$ is trivial.) (3) If X is a cohomology n-sphere, there is an integer k such that if $m = n-k(r+1)$, then A is a cohomology m-sphere. (4) If X is a locally orientable n-gcm (in the sense of Wilder), then A is a locally orientable m-gcm, m of the same form as above. Finally, (5) if X is a sphere-like n-gcm, then A is a sphere-like m-gcm. (Received December 12, 1957.)

542-142. Pentti Laasonen: On the difference methods in solving inhomogeneous elliptic differential equations.

In proving that the solution of an elliptic differential equation can be approximated arbitrarily close by a difference method one has to use either the character of the equation as an Euler equation or some explicit estimate for the truncation error. Neither of these methods can be applied directly if the inhomogeneous term is not sufficiently regular. In these cases the difference of the solutions of the differential equation and the difference equation often can be estimated by representing the solutions as an integral and as a sum, respectively, in terms of Green's functions and the inhomogeneous terms. For harmonic and biharmonic operators, it is possible to find explicit expressions for Green's functions at least in rectangular domains, and it turns out that this is sufficient to provide error estimates for much more complicated regions and, moreover, without irrelevant assumptions concerning the regularity of the inhomogeneous terms. (Received December 12, 1957.)

542-143. A. G. Anderson: The prediction of quantitative characteristics in polygenic systems. III.

A symbolic treatment is given of one phase of the method of prediction employed by the author in two earlier papers (Bull. Amer. Math. Soc. Abstracts 59-3-315 and 60-6-757). The plenary shapes of Etherington [On non-associative combinations, Proc. Royal Soc. Edinburgh vol. 59 (1939) pp. 153-162] are written, in terms of a generalization of the usual genetical meaning of F_n, as F_n in-

volving 2^n appropriate subscripts. Characteristics are then evaluated for certain pedegrees as functions of the various subscripts and their locations in the symbolic representation. (Received December 13, 1957.)

542-144. R. D. Anderson: The groups of all homeomorphisms of certain homogeneous spaces.

Let C be the Cantor ternary set and let U be the universal curve. Let G(C) and G(U) be the groups of all homeomorphisms of C and U respectively. It is shown by a rather easy argument that G(C) and G(U) are both (algebraically) simple. The techniques also can be used to show the simplicity of the group of all orientation-preserving homeomorphisms of the 2-sphere - a result previously asserted, these abstracts, Bull. Amer. Math. Soc. vol. 53 (1947) p. 506 by Ulam and von Neumann. (Received December 13, 1957.)

542-145. P. M. Anselone: Integral equations of radiative transfer.

Let $J(\tau)$, $\tau \geqq 0$, denote the unique non-negative, nontrivial, solution of the Schwarzchild-Milne integral equations, $J(\tau) = \int_0^\infty K_1(\tau-t)J(t)dt$, $F = 4\int_0^\infty K_2(\tau-t)\operatorname{sgn}(t-\tau)J(t)dt$, where $K_r(x) = 2^{-1}\int_0^1 e^{-|x|/\mu}\mu^{r-2}d\mu$ and $F > 0$. If the kernel functions are replaced by the nth order Gaussian numerical integration approximations, the resulting problem has a unique solution $J_n(\tau)$; this is a special case of a problem treated in the author's paper presented to the American Mathematical Society on October 21, 1957. The main theorem proved here is that $J_n(\tau) \longrightarrow J(\tau)$ uniformly on compact sets. This theorem was applied by the author (abstract in Amer. Math. Monthly vol. 64 (1957) p. 553) to the classical transfer problem for $I(\tau,\mu)$ and $J(\tau)$ defined by $\partial I/\partial\tau = I - J$, $J = 2^{-1}\int_{-1}^{1} I d\mu$, $F = 2\int_{-1}^{1} I\mu d\mu$, $I(0,\mu) = 0$ for $\mu < 0$, and $\lim_{\tau\to\infty} e^{-\tau/\mu} I(\tau,\mu) = 0$ for $\mu > 0$. It was shown that the sequence of Wick-Chandrasekhar approximations, $I_n(\tau,\mu)$, $n \geqq 1$, converges uniformly to $I(\tau,\mu)$ on compact sets. For details, see the author's 1957 Oregon State College doctoral dissertation. (Received December 13, 1957.)

542-146. B. J. Ball: The sum of two solid wild 2-spheres.

R. H. Bing proved (Ann. of Math. vol. 56 (1952) pp. 354-362) that the space obtained by sewing two solid horned spheres together along their boundaries in a certain fashion is homeomorphic to S^3. In this paper a wild 2-sphere, differing slightly from Alexander's horned sphere, is described, and it is shown that

two such spheres plus their boundaries can be sewn together along their boundaries in such a way that the resulting space in not S^3. (Received December 13, 1957.)

542-147. E. H. Bareiss: Modified Hessenberg method.

The Hessenberg Algorithm (R. Zurmühl, Praktische Mathematik, Springer, Berlin, 1957, p. 138 to 151) yields the characteristic equation for any square matrix in three major steps. The modified method achieves the same result in two steps. A check formula is also provided. All operations are of the same form, viz. evaluation of the scalar product of a row vector by a column vector. Working sheets will be available. If time permits several other algorithms will be compared. (Received December 13, 1957.)

542-148. F. G. Brauer and Shlomo Sternberg: Local uniqueness, existence in the large, and the convergence of successive approximations.

There is a close analogy between theorems assuring existence in the large of solutions of ordinary differential equations on the one hand, and local uniqueness theorems on the other. For example, Kamke's general uniqueness theorem corresponds to a theorem of Wintner on global existence. Furthermore, the hypotheses of all these uniqueness theorems suffice to prove the convergence of successive approximations. However, Müller has given an example of a differential equation with a unique solution for which the successive approximations do not converge. In this paper, a uniqueness theorem is proved which includes Kamke's theorem as a special case, and whose hypotheses are satisfied by Müller's example. Thus there can be no analogous theorem on convergence of successive approximations. However, a convergence theorem is proved which generalizes the analogue of Kamke's theorem. The duality between local uniqueness and global existence is explained by showing that global existence can be viewed as a sort of uniqueness at infinity. (Received December 13, 1957.)

542-149. C. W. Curtis: On the irreducible representations of certain Lie algebras of prime characteristic.

Let L be a Lie algebra over an algebraically closed field Ω of characteristic $p > 7$ whose Killing form is nondegenerate. Let H be a fixed Cartan subalgebra of L; $\alpha, \beta, \ldots$ the roots of L with respect to H; and $e_\alpha, e_\beta, \ldots$ the

corresponding root elements in L. The roots may be linearly ordered by a relation $<$. In this paper L is viewed as a restricted Lie algebra, and it is assumed that L is imbedded in its u-algebra U. U is a finite dimensional algebra, and consequently possesses at most a finite number of nonisomorphic irreducible right U-modules. Let M be an irreducible right U-module, and let $M_0 = \{x \mid x \in M, xe_\alpha = 0, \alpha > 0\}$. Then $M_0 \neq 0$, and there exists a weight λ of M such that $M_0 \subset M_\lambda$; λ is called the maximal weight of M. Two irreducible U-modules are U-isomorphic if and only if they have the same maximal weight. For every linear function λ on H such that $\lambda(h^p) = \lambda(h)^p$ whenever $h^p = h$ in H, there exists exactly one irreducible right U-module whose maximal weight is λ. As a corollary, it follows that the number of nonisomorphic irreducible right U-modules is exactly p^r, where r is the rank of L. (Received December 13, 1957.)

542-150. W. F. Eberlein: Integrals over function space. II. Preliminary report.

Integrals over the unit sphere S of ℓ_1 are investigated [Bull. Amer. Math. Soc. Abstract 63-6-675]. Although S is a compact Abelian semigroup under convolution, the ergodic invariant integral is degenerate: $I(f) = f(0)$, and the integrals of interest arise from consideration of cylinder functions and the Kolmogoroff-Banach compatibility conditions. The partial sums $s_n = s_n(x)$ of the power series $\sum_0^\infty x_n t^n$ absolutely convergent on the unit circle then form a martingale, and zero-or-one phenomena appear. In particular, if R(x) is the radius of convergence of the series, the measure is that induced by the "natural" integral, and e is the base of the natural logarithms, it turns out that -- THEOREM. $R(x) = e$ almost everywhere. (This research was supported in part by the Bureau of Ships and the Office of Naval Research). (Received December 13, 1957.)

542-151. Trevor Evans: Substitution algebras. I.

Some problems in many-valued logics lead to the study of the algebra of functions of one or more variables under the operation of substitution. This paper and subsequent papers are devoted to the study of algebraic systems, called substitution algebras, which are abstractions from these algebras of functions. In this paper the main emphasis is on the algebraic systems corresponding to the algebras of functions of one or two variables and of functions of

two variables only. A representation theorem is proved for such algebras, their automorphisms are studied and homomorphism theorems derived. Conditions are obtained for an element of the algebra to represent a quasigroup, group, semigroup, etc. (Received December 13, 1957.)

542-152. W. H. Fleming: Sets with finite perimeter. Preliminary report.

Let $\mathcal{F}$ denote the space of all summable functions f(x) on N-space R_N with compact support, such that the gradient of f in the distribution theory sense is a (vector) measure Φ. Let I(f) = total variation of Φ. A set E has finite perimeter if its characteristic function belongs to $\mathcal{F}$ [De Giorgi, Annali di Mat. vol. 36 (1954) pp. 191-213]. E is an elementary set if E is open and its frontier E^* is contained in the range of a Lipschitzian mapping with compact domain $\subset R_{N-1}$. H_N^k denotes Hausdorff k-measure in R_N. For any set E and $\delta > 0$ let $c_\delta(E) = \inf \sum H_N^{N-1}(E_i^*)$, where $E \subset \cup E_i$, E_i is elementary, and diam $E_i < \delta$. Let $c(E) = \lim_{\delta \to 0} c_\delta(E)$. For $N = 2$, $c(E) = 0$ iff $H_2^1(E) = 0$, but for $N > 2$ this question is open. A set E is called sharp if there exists a sequence E_n of elementary sets whose characteristic functions f_n tend to the characteristic function f of E c-almost everywhere such that $I(f_n - f) \to 0$. Theorem. If E has finite perimeter, then there is a sharp set E' differing from E in N-dimensional measure 0. E' is unique up to c- measure 0. The proof uses an approximation theorem for generalized surfaces [Fleming-Young, Rend. Circ. Mat. Palermo vol. 5 (1956) pp. 320-340; Theorem 4]. The sets of c-measure 0 are also the exceptional sets in the perfect functional completion [Aronszajn-Smith, Ann. Inst. Fourier Grenoble vol. 6 (1956) pp. 125-185] in the I-norm of the space of C^∞ functions in $\mathcal{F}$. (Received December 13, 1957.)

542-153. Jacob Feldman: Two bizarre connected spaces.

There exist: (1) A connected completely regular topological space X such that if f is any real-valued continuous function on X and α any real number, then $f^{-1}(\alpha)$ is the closure of its interior. (2) A connected compact Hausdorff space $\bar{X}$ such that if $\bar{f}$ is a continuous real-valued function on $\bar{X}$, and x is a point at which the value α is assumed but not a local extreme, then x is in the closure of the interior of $\{x | \bar{f}(x) = \alpha\}$. For $\bar{X}$ in (2), just take the Stone-Cech compactification of X in (1). (Received December 13, 1957.)

542-154. R. E. Fullerton: Support properties for cones with quasi interior points.

Let X be a linear topological space which is tonnelé. Let K be a closed convex cone in X with vertex $\mathcal{O}$. A point $x \in X$ is a quasi interior point if $K \cap (x - K)$ has its linear extension dense in X. All other points of K are called quasi boundary points of K. The following theorem is proved. Let K be a closed convex cone in X with nonvoid quasi interior K_q. Let x_0 be a quasi boundary point of K. Then there exist subsets H, S_1, S_2 of X with the following properties. (a) H is a proper linear subspace of X which contains x_0 but contains no quasi interior points of K and is maximal with respect to this property, (b) $H \cap S_1 = H \cap S_2 = S_1 \cap S_2 = \phi$, (c) $H \cup S_1 \cup S_2 = X$, (d) H, S_1, S_2 are convex, (e) $K_q \subset S_1$, (f) S_1 and S_2 are symmetric to H in the sense that if $x \in S$ and $y \in H$, there exists $z \in S_2$ with $y = (x + z)/2$, (g) Either H is a hyperplane or H, S_1, S_2 are all dense in X. Examples are given to show that either alternative in (g) can be satisfied. (Received December 13, 1957.)

542-155. J. W. Gaddum: Projective convexity in Euclidean space.

If x and y are two points in E_n, the set $\lambda x+(1 - \lambda)y$ is called the internal segment for $0 \leqq \lambda \leqq 1$, and the external segment for $\lambda \geqq 1$ or $\lambda \leqq 0$. A set A is called projectively convex if each two of its points determine a segment, either internal or external, lying in A. It is shown that the set of points of E_n at which a quadratic function is positive (non-negative, negative, non-positive) is projectively convex. Plane projectively convex sets are discussed. It is shown, among other things, that if A is a plane projectively convex set, either A or A' has property P_3. (See Valentine, Pacific J. of Math. vol. 7, no. 2). This and other properties are used to discuss projectively convex sets in E_n. (Received December 13, 1957.)

542-156. S. I. Goldberg: A theorem on the characterization of spheres.

In 1927 Cohn-Vossen established the result that two isometric analytic proper ovaloids are congruent. This proof has since been simplified by Shitomirsky (cf. H. Hopf, Selected topics in differential geometry in the large, New York University, 1955). It is shown that the formalism developed by Shitomirsky may be used to give a simple proof of the well-known fact that "a closed analytic surface S of genus zero and constant mean curvature H is a sphere." The proof of this result is easily generalized, so that the assumption

H = const. is not essential: Let $X = X(u,v)$ be the vector equation of the surface S where the u,v are coördinates of a point in a simply-connected domain of a parameter plane. Let $Edu^2 + 2Fdudv + Gdv^2$ and $Ldu^2 + 2Mdudv + Ndv^2$ be the first and second fundamental forms of S, resp. Then, if the condition H = const. is replaced by the differential equations: $H_vE - H_uF = a_1(L - HE) + a_2(M - HF) + a_3(N - HG)$ and $H_vF - H_uG = a_4(L - HE) + a_5(M - HF) + a_6(N - HG)$ where the a_i are arbitrary analytic functions, S is a sphere. (Received December 13, 1957.)

542-157. T. H. Hildebrandt: Systems of linear differentio-Stieltjes-integral equations.

The solution of the Stieltjes integral equation analogue of the integral equation equivalent of a system of linear differential equations: $dy_i/dx = \sum_{j=1}^{n} a_{ij}(x)y_j(x)$, $y_i(a) = c_i$, $a \leqq x \leqq b$, viz. $y_i(x) = \int_a^x \sum_{j=1}^{n} da_{ij}(t)y_j(t) + c_i$ follows the usual pattern for the case when the $a_{ij}(x)$ are continuous and of bounded variation (See H. S. Wall, Archiv der Math. vol. 5 (1954) pp. 160-167). When the $a_{ij}(x)$ have discontinuities the fact that the solutions $y_i(x)$ have discontinuities coincident with those of $a_{ij}(x)$ introduces difficulties. We assume $y_i(x)$ to be of bounded variation and evaluate the integral $\int_a^x da(t)y(t)$ as a Lebesgue Stieltjes integral. Then the Peano series need not converge, the solution of the system of equations requires conditions on the discontinuities of $a_{ij}(x)$ and involves terms of a product character. A fundamental system of solutions can however be expressed as a product integral $\int_a^x (I + dA)$, with suitable modifications at the points of discontinuity of the matrix A, limit of the approximating products being in the sense of successive subdivisions. (Received December 13, 1957.)

542-158. Guy Johnson, Jr.: A bound for the mass distribution of a function which has harmonic support.

Consider real valued functions $u(z)$ defined in a domain D of the complex plane. The function u is in (hs), the class of functions which have harmonic support in D, if and only if, for each z_0 in D, there is a function h harmonic and single valued in D such that $h(z_0) = u(z_0)$ and $h(z) \leqq u(z)$ in D. If u is in (hs) then it is subharmonic and is the logarithmic potential of a negative mass distribution $-\mu$ in D. Suppose u is in (hs) in $|z| < 1$, $u(z) < 1$ and $u(0) = 0$. If C_ρ denotes the set $|z| < \rho$, $0 < \rho < 1$, then $\mu(C_\rho) \leqq 2\rho/(1 - \rho)^2$. (Received December 13, 1957.)

542-159. Melvin Henriksen: The $a^{n(a)} = a$ theorem for semirings with commutative addition.

In the following, a semiring denotes an algebraic system with two binary operations (+) and (·) such that S(+) and S(·) are semigroups, S(+) is commutative and has an identity element 0 such that $0 \cdot a = a \cdot 0 = 0$ for all $a \in S$, and such that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for all $a,b,c \in S$. Consider the condition (*) for every $a \in S$, there is an integer $n(a) > 1$ such that $a^{n(a)} = a$. Theorem 1. If S is a division semiring (defined in the natural way), and (*) holds, then S is a commutative field, or S is a two-element lattice. Theorem 2. If S(·) has an identity element e, then S is a distributive lattice iff (*) holds, and $(e + a)^{-1}$ exists for all $a \in S$. These theorems can be generalized, but (*) alone does not imply that S is commutative; even if S is finite. (Received December 13, 1957.)

542-160. H. L. Hunzeker: The torsion of a circular shaft with an ellipsoidal cavity.

In the unpublished work of G. E. Hay the Torsion Problem for a shaft of circular cross sections is reduced to finding a Stress Function F of two curvilinear coordinates satisfying the basic differential equation $\nabla^2 F - 3r^{-1} \nabla r \cdot \nabla F = 0$ throughout a domain D of the shaft in a meridian plane. Here r is the distance of a point in D from the axis of revolution. Furthermore, F must be constant on the boundary of D as well as on the axis of revolution. It is assumed that a constant Twisting Couple is applied to the ends of the shaft which are far from that part of the shaft being considered and that other surfaces of the shaft are load free. In this paper the torsion of a circular shaft with a symmetrically located prolate ellipsoidal cavity is considered. The elliptic coordinates (ξ^1, ξ^2) of the meridian plane are related to rectangular cartesian coordinates (z,r) by $z = d \cosh \xi^1 \cos \xi^2$ and $r = d \sinh \xi^1 \sin \xi^2$ where d is one half the distance between foci. A Stress Function satisfying the basic differential equation and which satisfies the boundary conditions on the ellipse $\xi^1 = \xi_0^1$ and on the axis of revolution is obtained. The condition F = constant is satisfied on the outer boundary C of D for which numerical contours of C are computed. Normal and Shearing Stresses are computed. (Received December 13, 1957.)

542-161. W. B. Jurkat: On semi-groups of positive matrices. I.

The paper deals with the analytic properties of "generalized Markov processes", i.e. matrix-functions $P(t) = (p_{jk}(t))$ satisfying $P(t) \geqq 0$, $P(s + t)$

$= P(s)P(t)$ for $s, t > 0$ and $\lim_{t \to +0} P(t) = (\delta_{jk})$. The condition of row-sums being equal to 1 is omitted. Nevertheless many of the results of Austin, Chung, Doob, Kendall, Kolmogorov, Reuter and others extend to the general case. The proofs in many cases, however, have to be refined or changed. To mention two statements only: (i) $q_{jk} = \lim_{t \to +0} (p_{jk}(t) - \delta_{jk})/t$ exists, $-\infty < q_{jj} < +\infty$, $0 \leqq q_{jk} < +\infty$ $(j \neq k)$. (ii) If $q_{jj} < +\infty$ for fixed j, then for all k and $s, t > 0$ $p'_{jk}(s + t) = \sum_m p'_{jm}(s) p_{mk}(t)$, $p'_{kj}(s + t) = \sum_m p_{km}(s) p'_{mj}(t)$, the derivatives being continuous including the origin. (Received December 13, 1957.)

542-162. Naoki Kimura: Total orderings on a semilattice.

Let S be a semilattice. If there is a total ordering on S satisfying the monotonity condition: $a < b$ implies $ac \leqq bc$, then S is called orderable. The purpose of this note is to present the necessary and sufficient condition for a semilattice to be orderable. Let S be a semilattice. Then S is orderable if and only if it satisfies the following two conditions: (1) S does not contain four distinct elements a,b,c,d such that $ab = b$, $ac = c$ and $bc = d$. (2) S does not contain four distinct elements a, b, c, d such that $ab = ac = bc = d$. Let T be the set of all elements a such that $ab = a$ for some $b \neq a$. Then the cardinal of all different total orderings on S is 2^m, where m is the cardinal of T. A generalization of this result for a directed set and for an idempotent semigroup will also be presented. (Received December 13, 1957.)

542-163. Lee Lorch: The Lebesgue constants for the Borel and Euler summation of developments in Legendre polynomials.

It is shown that the nth Lebesgue constant for either the Borel or (classical) Euler summation of developments in terms of Legendre polynomials at the end-point $x = 1$ is $\Gamma(1/4) 2^{-1/4} \pi^{-3/2} n^{1/4} + O(1)$, as n becomes infinite. (In Borel's method, n is continuous, in Euler's, discrete.) Thus, there exist continuous functions for which these developments are not Borel (much less Euler) summable at the end-point. However, these methods are stronger than convergence when applied to such series, as they do sum the divergent Legendre developments of certain continuous functions. A continuous function constructed by Fejér (Math. Zeit. vol. 24 (1926) pp. 285-298) is shown here to have this property, even for the generalized (E,k) method for each positive k. This same divergent development, incidentally, is also shown to be (C,k)-summable for each $k > 0$, whereas, as is well known, (C,k)-summability of such expansions of

continuous functions is assured in general only for $k > 1/2$, even (C,1/2) being inadequate. (Received December 13, 1957.)

542-164. Marvin Marcus and Morris Newman: Permanents of doubly stochastic matrices.

The following problem was set by B. L. van der Waerden and is to be found in (Kŏnig, Theorie der Graphen. Chelsea, New York, 1950, p. 238): find the minimum value of the function p(S), where p denotes the permanent and S ranges over the polyhedron Π of all n-square doubly stochastic matrices. We obtain the following partial answers to the question: (i) If the minimum is attained in the relative interior of Π (those matrices in Π all of whose entries are positive) then it is assumed uniquely for the matrix J all of whose entries are 1/n; (ii) For $n = 3$ the minimum is assumed in the relative interior of Π; (iii) if the minimum is assumed at S and S is not in the relative interior of Π then not all the zeros of S occur in a fixed row. (Received December 13, 1957.)

542-165. Barth Pollak: Concerning a hermitian equation in a quaternion algebra.

Let Q_k be a quaternion algebra over a field k of characteristic $\neq 2$. If $x \in Q_k$ set $Sx = x + \bar{x}$ and $Nx = x\bar{x}$. Let $a \in Q_k$, $Sa = 0$ and $\sigma \in k^*$. Define $G(a,\sigma) = \{b \in Q_k | Nb = \sigma^2 Na, Sb = 0\}$ and $H(a,\sigma) = \{b \in Q_k | b = \bar{x}ax$ with $Nx = \sigma$ for some $x \in Q_k\}$. A criterion for membership in $H(a,\sigma)$ is given. A conjecture of Gordon Pall concerning a relationship between $H(a,\sigma)$ and $H(a,-\sigma)$ is established. From now on assume k is a local field and $Na \neq 0$. Then if Q_k splits, $H(a,\sigma) = H(a,-\sigma)$. However Q_k a division algebra implies $H(a,\sigma) \cap H(a,-\sigma) = \emptyset$ and $G(a,\sigma) = H(a,\sigma) \cup H(a,-\sigma)$. Now let M be a maximal order in Q_k and assume 2 is unramified. Let $a \in M$ and σ be a unit in k. Set $G(a,\sigma; M) = \{b \in M | Nb = \sigma^2 Na, Sb = 0\}$ and $H(a,\sigma;M) = \{b \in M | b = \bar{x}ax$ with $Nx = \sigma$ for some $x \in M\}$. A criterion for membership in $H(a,\sigma;M)$ is given. Let $Na \neq 0$ and 2 be a unit. If Q_k splits, $H(a,\sigma;M) = H(a,-\sigma;M)$. If Q_k is a division algebra, $H(a,\sigma;M) \cap H(a,-\sigma;M) = \emptyset$ and $G(a,\sigma;M) = H(a,\sigma;M) \cup H(a,-\sigma;M)$. (Received December 13, 1957.)

542-166. L. A. Rubel: A limit theorem for real functions.

THEOREM 1: Let $f(x)$ be a Lebesgue measurable real function defined for all $x > 0$ and satisfying (i) $\lim_{n\to\infty} f(n)$ exists and (ii) $\lim_{x\to\infty} f(x + a) - f(x) = 0$ for a set of $a > 0$ of positive measure. Then $\lim_{x\to\infty} f(x)$ exists. THEOREM 2: There is a function $f(x)$ satisfying all the conditions of the above theorem except measurability for which $\lim_{x\to\infty} f(x)$ does not exist. (Received December 13, 1957.)

542-167. Diran Sarafyan: Existence of singular points for a well known class of nonhomogeneous differential equations.

In a previous paper (Bull. Amer. Math. Soc. Abstract 63-2-256) the author has discussed the conditions under which the homogeneous differential equations $M(x,y)dx + N(x,y)dy = 0$ admit intersection or singular points. This paper deals with the singular points of the differential equations of the form $dy/dx = f((a_1x + b_1y + c_1)/(a_2x + b_2y + c_2))$ which can be readily made homogeneous provided $a_1b_2 - a_2b_1 \neq 0$. Henceforth it will be assumed that this requirement is satisfied. The author shows that the existence of singular points for the first differential equation above is related to the existence of real valued solutions of the equation $r = f((a_1 + b_1r)/(a_2 + b_2r))$. If the latter equation has no real solution then the differential equation $dy/dx = f((a_1x + b_1y + c_1)/(a_2x + b_2y + c_2))$ has no singular points. In particular, if the differential equation is of the form $dy/dx = (a_1x + b_1y + c_1)/(a_2x + b_2y + c_2)$ then the functional equation is $r = (a_1 + b_1r)/(a_2 + b_2r)$. It follows that in this case there will be no singular points if $(a_2 - b_1)^2 + 4a_1b_2 < 0$. (Received December 13, 1957.)

542-168. Albert Schild: On a class of star shaped mappings.

In a paper by R. F. Gabriel (Proc. Amer. Math. Soc. February 1955) a class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ is mentioned, with the property $\mathrm{Re}(zf'(z)/f(z)) \geqq 1/2$ for $|z| < 1$. In the present paper this class of functions is investigated further, both from an analytic and geometric point of view. In particular, methods are established for constructing functions with the above property. Also, for special subclasses of the class of functions $w = f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ schlicht in the unit circle, constants r are found such that $|z| \leqq r$ will be mapped by $w = f(z)$ into a region for which $\mathrm{Re}(zf'(z)/f(z)) \geqq 1/2$. The coefficient problem for this class of functions is also considered. Elementary classical methods of schlicht functions are employed in this paper. (Received December 13, 1957.)

542-169. J. R. Shoenfield: Degrees of decidability of formal theories.

The following results answer questions proposed by Feferman (J. Symbolic Logic, vol. 22 (1957) pp. 161-175). Theorem 1: If A is a recursively enumerable, nonrecursive set, there is an essentially undecidable theory with a finite number of constants in which the class of theorems is of the same degree of undecidability as A. The proof requires the following lemma: If A is a recursively enumerable, nonrecursive set, then there are recursively inseparable sets B and C which are recursive in A. Theorem 2: If K is a theory in which the function U and the predicate T are representable, then K is essentially creative, provided that the symbol T' representing T satisfies the following uniqueness condition: for every e and n, $(T(\Delta_e, \Delta_n, x)$ and $T(\Delta_e, \Delta_n, y)) \Rightarrow x = y$ is provable in K. In particular, the system of Kreisel cited in Feferman's paper satisfies the conditions of Theorem 2. (Received December 13, 1957.)

542-170. R. J. Silverman and Ti Yen: Characteristic functionals.

Let E be a partially ordered linear normed space over the real numbers whose positive cone K has nonempty interior K^0. A functional f in E^* is called positive if $f(K) \geq 0$, $f \neq 0$. Let Γ be a semi-group of strictly positive linear operators on E (i.e. $\Gamma(K^0) \subset K^0$). It is known that if Γ is commutative, then Γ has characteristic positive functionals f. (i.e. $A^*f = \lambda_A f, \lambda_A > 0, A \in \Gamma$). (1) This theorem does not carry over in general to left solvable semi-group. However, if $\Gamma^{(1)}$ is a left commutator subsemi-group Γ (i.e. for any $A, A' \in \Gamma$ there exists $A'' \in \Gamma^{(1)}$ such that $AA' = A''A'A$), then if $\Gamma^{(1)}$ has an invariant positive functional, Γ has a characteristic positive functional. Also if Γ, considered as an abstract semi-group, has a right invariant mean, then the following statements are equivalent. (a) Γ has a positive functional. (b) There exists a closed hyperplane $X \subset E$ such that $X \cap K^0 = \emptyset$ and $\Gamma(X) \subset X$. (c) There exists a positive functional f such that $f(AA'u) = f(Au)f(A'u)$, $A, A' \in \Gamma$ for some $u \in K^0$. (d) There exists a positive functional f such that $f(AA'A''u) = f(A'AA''u)$, $A, A, A'' \in \Gamma$ for some $u \in K^0$. (e) There exist a homomorphism $\lambda: A \to \lambda_A$ of Γ into the multiplication group of the real numbers, two positive numbers m and M and a positive functional f such that $m\lambda_A \leq f(Au) \leq M\lambda_A$, $A \in \Gamma$, for some $u \in K^0$. (Received December 13, 1957.)

542-171. R. J. Silverman and Ti Yen: The Hahn-Banach extension theorem.

Let V be a partially ordered linear space over the real numbers with positive cone K. Then V has the Hahn-Banach Extension property if for every quadruple [Y,p,X,f] where (1) Y is a linear space, (2) p is a positive homogeneous subadditive function from Y to V, (3) X is a subspace of Y, (4) f is a linear function from X to V such that $f(x) \leqq p(x), x \in X$, implies there exists a linear extension F of f to all of Y such that $F(y) \leqq p(y), y \in Y$. The space V has the least upper bound property if every set in V bounded from above has a least upper bound. The space V is lineally closed if every line in V intersects K in a closed set in the line (or an empty set). There is an example of a two dimensional partially ordered space with the Hahn-Banach extension property which does not have the least upper bound property. However, Theorem: the space V has the Hahn-Banach extension property and is lineally closed, if and only if V has the least upper bound property. (Received December 13, 1957.)

542-172. D. W. Wall: A characterization of generalized uniserial algebras.

Let A be a finite dimensional algebra over a field. THEOREM 1. If A is UMFR (QF-3) but is not QF-2 then there exists a two-sided ideal Z of A such that A/Z is not UMFR. Proof. The proof makes use of, and follows the same pattern as, an earlier result (Trans. Amer. Math. Soc. (to appear)) of which this is a generalization. Also used are the characterizations of the subclasses of the UMFR algebras in terms of the socles of the primitive ideals (Duke Math. J. (to appear)). First, make a suitable choice of a weakly subordinate primitive ideal L. Second, construct a two-sided ideal Z which will contain certain minimal subideals and such that $L \cap Z = 0$. Thus, L can be considered as a primitive ideal of A/Z and by consideration of the possible cases it can be shown that L cannot be weakly subordinate to any set of dominant ideals in A/Z. THEOREM 2. An algebra A is generalized uniserial if and only if for every Z, A/Z is UMFR. Proof. This follows by combining Theorem 1 and an earlier result which characterized the generalized uniserial algebras as those algebras A such that all A/Z are QF-2. (Received December 13, 1957.)

542-173. Paul Weiss: The initial value problem of nth order linear differential and difference equations with constant coefficients.

Let $P_n(s) = s^n + a_1 s^{n-1} + \ldots + a_n$ be a polynomial of nth degree with roots $s_1, s_2, \ldots, s_n$. Let $Q_k(s)$ be the sequence of associated "Horner" polynomials of

degree k, $(k = 0,1,\ldots,n)$, defined recursively by $Q_0(s) = 1$, $Q_k(s) = s \cdot Q_{k-1}(s) + a_k$ for $0 < k \leqq n$, so that $Q_n(s) = P_n(s)$. Let h_i $(i = 0,1,\ldots)$ be the set of Jacobi's complete symmetric functions of $s_1,\ldots,s_n$, these being expressible in terms of the coefficients a_k of $P_n(s)$ through their generating function $s^n \cdot P_n^{-1}(s) = \sum_{i=0}^{\infty} h_i s^{-i}$. Then the solution $u(t)$ of the differential equation $P_n(D)u(t) = f(t)$, $D = d/dt$, $0 \leqq t < \infty$, which, for $t = 0$, satisfies the initial conditions $D^r u = v_r$, $(r = 0,1,\ldots,n-1)$, reads $u(t) = \sum_{r=0}^{n-1} v_r \cdot Q_{n-1-r}(D)F(t) + \int_0^t F(t-t')f(t')\,dt'$, where $F(t)$ is the solution of the homogeneous equation with initial values $v_r = \delta_{r,n-1}$; it is the $(n-1)$-st divided difference of $\exp(st)$ with respect to $(s_1,s_2,\ldots,s_n)$ and it reads $F(t) = \sum_{i=0}^{\infty} h_i t^{n-1+i}/(n-1+i)!$. Similarly, the solution $u(m)$ of the difference equation $P_n(E)u(m) = f(m)$, $Eg(m) = g(m+1)$, $m = 0,1,\ldots$, which satisfies the initial conditions $u(r) = v_r$, reads $u(m) = \sum_{r=0}^{n-1} v_r \cdot Q_{n-1-r}(E)S(m) + \sum_{k=0}^{m-1} S(m-1-k)f(k)$, where $S(m) = h_{m-n+1}$. (Received December 13, 1957.)

542-174. R. E. Zink: On regular measures and Baire functions.

Let X be a topological space, and let $(X, \underset{\sim}{S}, \mu)$ be an associated measure space. Let f be a non-negative measurable ($\underset{\sim}{S}$) function, and let the measure ν be defined on S by means of the equation $\nu(E) = \int_E f(x)d\mu(x)$. It has been shown by this writer [Duke Math. J. vol. 24 (1957) pp. 127-135] that, if μ is outer regular, then the necessary and sufficient conditions for the outer regularity of ν are: (1) if f is integrable over E, there exists a sequence of measurable open sets $\{U_n\}$ such that E is contained in $\bigcup_{n=1}^{\infty} U_n$ and f is integrable over each U_n; (2) if f is integrable over E and ϵ and δ are any positive numbers, there exists a measurable open set V containing $E \cap \{x: f(x) = 0\}$ such that $\mu(\{x: f(x) > \delta\} \cap V) < \epsilon$. Now the second condition appears to be a sort of continuity restriction, and, in fact, every continuous measurable function does satisfy this requirement. By using the time-honored techniques of measure theory, one can establish the following theorem: If $\underset{\sim}{S}$ is the Borel σ-ring (i.e. the σ-ring generated by the class of all compact subsets of X), then every measurable Baire function satisfying (1) also satisfies (2), and, as a result, the corresponding measure ν is outer regular. (Received December 13, 1957.)

542-175. Jörg Mayer-Kalkschmidt: Conditional inclusion of summability methods.

Consider the space of sequences $\{s_n\}$ and two matrix transformations

$(a_{n\nu})$ and $(b_{n\nu})$; let $\Omega(n)$ be a monotonic function defined for all positive integers. We denote the set of sequences $\{s_n\}$ for which $\tau_n = \sum_{\nu=0}^{\infty} a_{n\nu} s_\nu = O[\Omega(n)]$ as $n \to \infty$ by $[A,O(\Omega)]$. A matrix $(c_{n\nu})$ has the property $(\mathcal{M})$, if it is regular, triangular and if $|\sum_{\nu=0}^{n} c_{m\nu} s_\nu| \leqq K |\sum_{\nu=0}^{n'} c_{n'\nu} s_\nu|$ where $n' \leqq n \leqq m$. By $(c'_{n\nu})$ we shall signify the inverse of the matrix $(c_{n\nu})$. The following theorems are proved: (1) If $(a_{n\nu})$ has the property $(\mathcal{M})$, $(b_{n\nu})$ is triangular and if $\sum_{\nu=0}^{n} \Omega(\nu) |\Delta_\nu b_{n\nu}/a_{n\nu}| = O(1)$, then $[A,O(\Omega)] \subset [B,O(1)]$, where $\Omega(n)$ is monotonically increasing. (2) If $(b_{n\nu})$ is regular and the inverse of $(a_{n\nu})$ exists and if $a'_{n\nu} \to 0$ as $n \to \infty$ then $\sum_{\nu=0}^{\infty} \Omega(\nu) |\sum_{\mu=0}^{\infty} b_{n\mu} a'_{\mu\nu}| = O(1)$ is necessary and sufficient for $[A,O(\Omega)] \subset [B,O(1)]$. Several variations and extensions of these theorems are possible. We mention two immediate consequences, $[(C,1), O(n^{-1/2})] \subset [E,O(1)]$ and $[(C,k'),O(n^{k-k'})] \subset [(C,k),O(1)]$ where $k < k'$. (Received December 13, 1957.)

542-176. Avron Douglis: On a boundary problem for a quasi-linear parabolic equation.

Estimates are obtained which pertain to the solution $u(x,t)$ of a certain kind of boundary problem for a quasi-linear parabolic equation of the form $f(x,t,u)u_t = u_{xx}$ ($f \geqq a > 0$). The domain considered is a region of the form U: $0 < x < y(t)$, $0 < t \leqq T$, the right side of which is an increasing, continuous curve C: $x = y(t)$ issuing from the origin such that $|y(t_1) - y(t_2)| \leqq |t_1 - t_2|$. Both u and u_x are assumed to be uniformly continuous in U and to satisfy the boundary conditions $u_x(0,t) = -1$, $u(y(t),t) = 0$. Furthermore, f is assumed to have a sufficient number of continuous derivatives for (x,t) in U and u a sufficient number of derivatives in U and on the left boundary $x = 0$. Bounds are then obtained for u and its derivatives in any finite subregion of U not abutting upon C. The bound for u_x, already known from Dr. Jim Douglas's work, is uniform over U, but the second and higher derivatives of u at a point P of U behave like certain reciprocal powers of the distance from P to C. The results are of interest in connection with heat conduction problems involving a free boundary between two phases of the conducting substance. (Received December 13, 1957.)

542-177. W. T. Guy, Jr.: Approximations for the inverse Laplace transform.

Certain approximations were presented for the inverse Laplace transform. These approximations were illustrated by several special cases. (Received December 13, 1957.)

542-178. R. L. SanSoucie: Sequences with property G.

A sequence $\{a_n\}$ of integers is said to have property G if and only if the following conditions hold: (1) $a_n = +1$ for $n = 1,2,\ldots$; (2) $\{a_n\}$ is periodic of period r; (3) $\sum_{n=1}^{r} a_n = \pm 1$ and (4) $\sum_{n=1}^{r} a_n a_{n+t} = -1$ for integral t, $1 \leqq t \leqq r - 1$. The following results on sequences with property G are among those proved. I. If $\{a_n\}$ has property G with period r, then $r = 4m - 1$ for some m. II. If $r = 4m - 1$, then there exists a sequence with property G having period r if and only if there exists a (Hadamard) difference set mod r; that is, if and only if the (v,k,λ) problem, for $v = 4m - 1$, $k = 2m - 1$, $\lambda = m - 1$, has a cyclic solution in the sense of Hall and Ryser. III. There exists a Hadamard difference set mod $v = 2^n - 1$ for every positive integer n. (The proof uses an unpublished result by S. W. Golomb on sequences with property G.) It follows from Theorem 2.1 of Marshall Hall [Proc. Amer. Math. Soc. vol. 7 (1956) pp. 975-986] that 2^s, for $s = 1,2,\ldots, n - 1$, is a multiplier of these difference sets. Conjecture: 4 and m are always multipliers for Hadamard difference sets mod $4m - 1$. (Received December 13, 1957.)

542-179. D. W. Robinson: A note on diagonable matrix polynomials.

Let ϕ be a polynomial and A a square matrix. L. Mirsky and R. Rado (A note on matrix polynomials, Quart. J. Math., Oxford vol. 8 (1957) pp. 128-132) have recently given sufficient conditions so that if $\phi(A)$ is diagonable, so is A. This note provides necessary and sufficient conditions for $\phi(A)$ to be diagonable, and gives the result of Mirsky and Rado as a corollary. Specifically, over an algebraically closed field of characteristic zero, the following theorem is demonstrated: Let A be an m-by-n matrix with distinct eigenvalues $\alpha_1,\ldots,\alpha_r$ of respective multiplicities $s_1,\ldots,s_r$ in the minimum polynomial of A. Let ϕ be any polynomial. Then $\phi(A)$ is diagonable if and only if $\phi^{(m)}(\alpha_i) = 0$ for all α_i and m such that $1 \leqq m < s_i$. (Received December 13, 1957.)

542-180. B. J. Ball: Certain collections of arcs in E^3.

Suppose α and β are horizontal planes in E^3 and G is a continuous collection of mutually exclusive arcs such that (1) each element of G has one end point on α and the other on β, (2) no horizontal plane contains two points of an element of G and (3) G^*, the sum of the elements of G, is compact and every component of G^* is an element of G. It does not follow that the decomposition space whose nondegenerate elements are the arcs of G is homeomorphic to E^3 (see R. H. Bing, Ann. of Math. vol. 65 (1957) pp. 484-499) or, even if this decomposition space is E^3 and each element of G is a straight line interval, that G is topologically equivalent to a collection of vertical intervals. In this paper a condition is given which is necessary and sufficient for the existence of a homeomorphism of E^3 onto itself which leaves each point of α fixed, does not change the z-coordinate of any point and takes each element of G onto a vertical interval. (Received December 13, 1957.)

542-181. C. W. Curtis: Quasi-Frobenius rings and Galois theory.

The following theorem is proved. Let M be a finitely generated, faithful, projective left module for a quasi-Frobenius ring R. Then M is also a projective, finitely generated $\mathrm{Hom}_R(M,M)$-module, and $\mathrm{Hom}_R(M,M)$ is a quasi-Frobenius ring. An example shows that if M is not projective, then $\mathrm{Hom}_R(M,M)$ may not be quasi-Frobenius. By means of the theorem and some results of Kasch (Math. Ann. vol. 127 (1954) pp. 453-474), certain applications to the Galois theory of simple rings are obtained. Let A be the full ring of linear transformations on a finite dimensional vector space M over a division ring Δ, and let G be a group of automorphisms of A of finite reduced order. Sufficient conditions are derived in order that the fixed ring I(G) of elements in A left fixed by all the automorphisms in G be a quasi-Frobenius ring, and M a projective I(G)-module. If these conditions are assumed, then it is also possible to derive a formula which expresses the reduced order of G in terms of the height and index relative to A of the indecomposable right ideal direct components of I(G). (Received December 13, 1957.)

542-182. Melvin Henriksen: Some remarks on a paper of Aronszajn and Panitchpakdi.

In the paper of the title (Pacific J. Math. vol. 6 (1956) pp. 405-439), a number of problems are posed. Problem 2 is answered in the affirmative,

while Problem 3 is answered in the negative. The solutions (which are both examples) are obtained by first showing that a normal space satisfies ($Q_{\aleph_1}$) in the sense of Aronszajn and Panitchpakdi iff it is an F-space in the sense of L. Gillman and the author (Trans. Amer. Math. Soc. vol. 82 (1956) pp. 366-391), and then utilizing results in this latter paper. (Received December 13, 1957.)

542-183. Melvin Henriksen: Ideals in semirings with commutative addition.

Let S denote a semiring in the sense of the preceeding abstract. An ideal I of S is a nonempty subset I of S such that a, $b \in I$, and $x \in S$ imply that $a + b$, xa, and ax are in I. A k-ideal I of S is an ideal of S such that $x, x + y \in I$ imply that $y \in S$. A subset I of S is the kernel of a homomorphism iff I is a k-ideal. A ring without an identity element may contain ideals in the above sense that are not ideals in the ring-theoretic sense. The k-ideals of S (partially ordered by set inclusion) form a modular lattice, while (as is known) the lattice of ideals of S need not be modular. If $S(\cdot)$ has an identity element, then every proper k-ideal of S is contained in a maximal (proper) k-ideal. For background, see S. Bourne, (Proc. Nat. Acad. Sci. U. S. A. vol. 42 (1956) pp. 632-638). (Received December 13, 1957.)

542-184. Naoki Kimura: On some special idempotent semigroups (III).

If an idempotent semigroup satisfies the following identity: (1) $axy = axay$ ($xya = xaya$), (2) $axy = axyay$ ($xya = xaxya$), (3) $axy = axyayxy$ ($xya = xyxaxya$), it is called (1) right (left) quasi-normal, (2) right (left) semi-normal, (3) right (left) semi-regular, respectively. The purpose of this note is to present the structure theorems of these idempotent semigroups. Let $\mathcal{P}(\mathcal{O})$ be the equivalence on an idempotent semigroup S defined by $x \mathcal{P} y$ if and only if $xy = x$ and $yx = y$ ($x \mathcal{O} y$ if and only if $xy = y$ and $yx = x$). Then (i) $\mathcal{P}(\mathcal{O})$ is a congruence on S if and only if it is left (right) semi-regular. Further in this case $S/\mathcal{P}$ ($S/\mathcal{O}$) is a left (right) regular idempotent semigroup. (ii) $\mathcal{P}(\mathcal{O})$ is a congruence on S and $S/\mathcal{P}$ ($S/\mathcal{O}$) is a left (right) normal idempotent semigroup if and only if it is left semi-normal. (iii) Both $\mathcal{P}$ and $\mathcal{O}$ are congruences on S and $S/\mathcal{P}$ ($S/\mathcal{O}$) is left (right) normal, if and only if S is left (right) quasi-normal. Further in this case S is the spined product of $S/\mathcal{P}$ and $S/\mathcal{O}$. As applications of these we can prove the structure theorems for a regular idempotent semi-group and for a normal idempotent semigroup. (Received December 13, 1957).

542-185. Pasquale Porcelli: On the reduction of weak convergence.

Let X be a set, G(X) be the Banach space of all bounded and real valued functions on X where for each f in G(X) $\|f\|$ (norm of f) = $\mathrm{LUB}_{x\in X}|f(x)|$, and G*(X) the first conjugate space of G(X). THEOREM: If G'(X) is a separable subspace of G(X), then there exist a set T and a separable subspace G'(T) of G(T) such that, (1) G'(X) and G'(T) are isomorphic and isometric, (2) T is a linearly ordered set with a max and a min, say $\bar{t}$ and $\underline{t}$, (3) T has the LUB property, (4) if b is in G'(T), then b is continuous on T, (5) if L is in G*(T), then there exists a function π of bounded variation on T (i.e. $\exists M > 0$ such that $\sum_{i=0}^{m-1}|\pi(t_{i+1})-\pi(t_i)| \leqq M$, $\underline{t} = t_0, \bar{t} = t_m$, and $\{t_i\}_{i=0}^{m-1}$ any subdivision of T) such that $L(b) = \int_{\underline{t}}^{\bar{t}} b\,d\pi$ for each b in G'(T) where the integral is in the Pollard-Stieltjes sense, and (6) if $\{b_n\}_{n=1}^{\infty}$ is a sequence in G'(T), then $b_n \longrightarrow 0$ (weakly) if, and only if, $b_n(t) \longrightarrow 0$ for each t in T and $\exists M > 0$ such that $\|b_n\| < M$ for $n = 1,2,\ldots$. One import of the theorem is that it makes it possible to reduce the study of weak convergence in general function spaces to the case where the elements of the conjugate space are represented by infinitely additive measures. (Received December 13, 1957.)

542-186. J. R. Shoenfield: The class of recursive functions.

The predicate "α is recursive" can be expressed in the form (Ex)(y)(Ez)R(α,x,y,z), but not in the dual form (x)(Ey)(z)R(α,x,y,z), with R a recursive predicate. The main tool in the proof is Baire's theorem applied to the space of all functions from the natural numbers to the natural numbers. (Received December 13, 1957.)

542-187. Alexander Weinstein: On the heat equation with subharmonic initial data.

Let $v(x_1,x_2,\ldots,x_m,t)$, $t > 0, -\infty < x_i < \infty$, denote the classical Poisson-Fourier solution of the heat equation $v_t = \Delta v$. Let the initial value of v for $t = 0$ be a subharmonic function $f(x_1,x_2,\ldots,x_m)$. Then it is proved that v is an increasing function of t and that v is convex in $t^{(2-m)/2}$ for $m \neq 2$ and convex in log t for $m = 2$. Moreover $t^{\alpha}v$ is convex in t^{α} for $\alpha \geqq (m-2)/2$. (Received December 13, 1957.)

542-188. Ichiro Amemiya and Israel Halperin: Finiteness and continuity in complemented modular lattices.

Kaplansky has made the remarkable discovery that every orthocomplemented complete modular lattice is a continuous geometry. His proof of this geometric theorem is essentially algebraic and requires (i) an analysis of the centre and (ii) the deep coordinatization theorem of von Neumann. We give a simple geometric proof which avoids both (i) and (ii). Let L be a complemented $\aleph_0$-complete modular lattice. We prove: (α) L is finite if (*) for each nonzero b there is a dual automorphism ϕ such that a nonzero part of b is perspective to some element disjoint from ϕ(b) ((*) certainly holds if L is orthocomplemented). (β) For every infinite cardinal $\aleph$, L is $\aleph$-continuous if (i) it is $\aleph$-complete, (ii) it is finite, and (iii) every nonzero b contains a nonzero b_1 such that b_1 has a certain property (S). (Received December 16, 1957.)

542-189. R. A. Beaumont: Generalized rings.

An additive abelian group R, closed with respect to multiplication is called an (m,n)-distributive ring if the generalized distributive law, $(\sum_{i=1}^{m} a_i)(\sum_{j=1}^{n} b_j) = \sum_{i=1,j=1}^{m,n} a_i b_j$ is satisfied for fixed integers $m \geqq 2$, $n \geqq 2$, and for all a_i, $b_j \in R$. If I is an ideal in R, then the usual construction yields a residue class (m,n)-distributive ring R - I. Moreover, it is proved that R - I is an ordinary ring, and in particular R - (0) is a ring, where (0) is the ideal generated by the additive identity. The mapping $R \longrightarrow R - (0)$ is a mapping of all proper (m,n)-generalized rings onto the set of all rings. The following structure theorem is obtained: Let R be an associative (m,n)-distributive ring such that $0 \cdot 0 = 0$ and 0 is the center of R. Then R is isomorphic to a subdirect sum of R - (0) and (0). (Received December 16, 1957.)

542-190. Richard Block: Relations between torsion-free abelian groups and certain Lie algebras.

Some modified group algebras which are analogs of Lie algebras of characteristic p are considered. Let $G \neq 0$ be a torsion-free abelian group, F a field of characteristic 0, g an additive and f a biadditive mapping of G (resp. $G \times G$) into F. Let $L = L(G,g,f)$ be the (infinite-dimensional) algebra with basis elements u_α indexed by G and with multiplication $u_\alpha u_\beta = [f(\alpha,\beta) + g(\alpha - \beta)]u_{\alpha+\beta}$. Assume that $g \not\equiv 0$, and that there is an additive mapping h of G into F such that for all α, β in G, $f(\alpha,\beta) = g(\alpha)h(\beta) - g(\beta)h(\alpha)$ (a n.a.s.c. for L to be a Lie alge-

bra). Then L is simple if and only if there is no $\delta = 0$ in G for which $g(\delta) = 0$ and $h(\delta) = 0$ or 2. Under these conditions the derivations are determined (the algebra of outer derivations is finite-dimensional if G has finite rank); using them it is proved that an isomorphism of L(G,g,f) onto an algebra L(G',g',f') induces an isomorphism $\alpha \to \alpha'$ of G onto G' for which there is a scalar c such that for all α,β in G, $g'(\alpha') = cg(\alpha)$ and $f'(\alpha',\beta') = cf(\alpha,\beta)$, and the automorphisms of L(G,g,f) are determined in terms of those of G. The analogs of the characteristic p algebras L(G,g,f) of the author [forthcoming in Trans. Amer. Math. Soc.], which involve a decomposition of G, are also considered, and theorems similar to the above are proved by extending the author's previous determination of derivations to the characteristic 0 case. (Received December 16, 1957.)

542-191. F. J. Bureau and L. R. Bragg: The solutions of a system of partial differential equations in a region of hyperbolic normality. Preliminary report.

This paper is concerned with obtaining representations for the solutions of a system of second order partial differential equations in a region in which each equation is of the hyperbolic normal type. The system is reduced to a matrix equation to which is applied the method of singularities. The representations are inferred from the solutions of the adjoint equation and are expressed in terms of finite and logarithmic parts of divergent integrals. The authors first treat the equation with analytic coefficients, then minimize the number of derivatives required of the coefficients to yield representations by this method. (Received December 16, 1957.)

542-192. Edgar Enochs: p-closed groups.

Let G be a primary reduced group of type 1. Consider the topology induced on G by letting the subgroups p^nG be a fundamental system of neighborhoods of the identity. Such a group is said to be p-closed if and only if every series formed from a null sequence of bounded order converges. Then it is proved that: (1) Given a basis subgroup B of G, and a direct decomposition of B, $B = \sum_{\nu \in I} B_\nu$, there exists at most one decomposition of G, $G = \sum_{\nu \in I} G_\nu$, such that $G_\nu \subset B_\nu$ for all $\nu \in I$. (2) G is p-closed if and only if for every basis subgroup B and every finite decomposition of B, $B = \sum_{i=1}^{n} B_i$, there exists a decomposition of G, $G = \sum_{i=1}^{n} G_i$, such that $G_i \supset B_i$. (3) If G contains ele-

ments of arbitrary large order, then G has a basis B, admitting an enumerable decomposition, $B = \sum_{i=1}^{\infty} B_i$, such that there is no enumerable decomposition of G, $G = \sum_{i=1}^{\infty} G_i$, such that $G_i \supset B_i$ for all i. (Received December 16, 1957.)

542-193. Murray Gerstenhaber: On deformations of compact complex manifolds.

Let V be a compact complex analytic manifold, S the sheaf of germs of holomorphic tangent vector fields on V and θ an element of $H^1(V,S)$. Necessary conditions are given for θ to be associated with a one-parameter family of deformations of V. In an equivalent formulation of the problem, it is shown that for a "nondefective" one-parameter family of deformations of V, i.e., one in which the deformation essentially is nowhere confined to a manifold of lower dimension, a necessary and sufficient condition that $\omega = \sum a_{\alpha\bar{\beta}}(\partial/\partial z^{\alpha})(d\bar{z}^{\beta})$ be a tensor variation of complex structure is that the Poisson bracket $[\omega, \text{periods of } \omega] = 0$, for suitably defined "periods" of ω, which are tangent vector fields on V. (Received December 16, 1957.)

542-194. Michael Golomb: Existence and expansion theorems for periodic and almost periodic solutions of nonlinear differential systems.

A novel method for finding solutions of nonlinear differential systems possessing absolutely convergent Fourier expansions (not necessarily harmonic) is described. The method was originated by Wasow and is applied here to systems which are analytic in the dependent variables and have absolutely convergent Fourier expansions in the independent variable. The coefficients in the expansion of the solution are found by solving a recursive system of linear algebraic equations. Under special conditions the particular solution found is imbedded in a field of solutions with similar expansions and is proved to be uniformly and asymptotically stable. Perturbations of the nonlinear system are also considered including the degenerate case (vanishing Jacobian). (Received December 16, 1957.)

542-195. Henry Helson and D. B. Lowdenslager: Prediction theory for vector valued processes.

Let W be a Borel measure on the unit circumference, with values finite positive semi-definite matrices, and let it have absolutely continuous part with derivative M. Then $\inf(2\pi)^{-1}\int_0^{2\pi} \operatorname{tr}\{(A_0 + A(\theta))^*(A_0 + A(\theta))dW(\theta)\}$

$= \exp\{(2\pi)^{-1}\int_0^{2\pi} \log \det(M(\theta))d\theta\}$, where the infimum is taken over all matrices A_0 of determinant 1 and trigonometric polynomials $A(\theta) = \sum_{n>0} A_n e^{in\theta}$ with matrix coefficients, and where tr and det denote the trace and determinant. The authors use this Theorem to obtain information on the structure of vector valued processes. The proofs extend to the case where the group of the integers is replaced by a totally ordered abelian group, and the results obtained generalize Theorems of F. and M. Riesz and Bochner, Wiener, and Beurling. (Received December 16, 1957.)

542-196. Bruce Kellogg: Hyperbolic equations with multiple characteristics.

Hyperbolic equations in two independent variables with multiple characteristics and variable coefficients of the type considered by A. Lax (Comm. Pure Appl. Math. vol. 9 (1956) pp. 135-169) are treated by the method of Garding (Cauchy's Problem for Hyperbolic Equations, lecture notes, University of Chicago, 1957). A Friedrichs-Lewy inequality and some dual inequalities are obtained, a theory of partial adjoints is constructed, and the Cauchy problem is solved for a general class of initial data including distributions. The results are expressed in terms of some new norms which involve directional derivatives of a function in the characteristic directions of a hyperbolic operator. (Received December 16, 1957.)

542-197. C. W. McArthur: On totally bounded sets in sequence spaces.

For a Banach space X let U denote those sequences $s = \{x_i\}$ in X such that $\sum_{i=1}^{\infty} x(i)$ converges unconditionally. With $\|s\| = \sup\{\|\sum_{i\in F} x(i)\|: F \text{ finite}\}$, U is a Banach space. For each natural number n define T_n on U to itself by $T_n(s) = s_n$ where $s_n(i) = s(i)$ if $1 \leq i \leq n$ and $s_n(i) = \theta$ if $i > n$. It is shown that a subset S of U is totally bounded if and only if $\lim_n T_n(s) = s$ uniformly on S and $\{s(i): s \in S\}$ is totally bounded in X for each i. This result and several known similar results are shown to be special cases of a more general theorem which gives necessary and sufficient conditions for total boundedness in a certain type of Banach space. (Received December 16, 1957.)

542-198. L. F. McAuley: Some upper semi-continuous decompositions of Euclidean 3-space into itself.

This paper gives several u.s.c. decompositions of E^3 which are E^3. One is the following. Suppose that G is an u.s.c. collection of straight line intervals and points filling up E^3 such that (1) the sum of the nondegenerate elements of G is a G_δ set and (2) there exists a sequence $\{P_i\}$ of planes such that a nondegenerate element of G intersects P_i if and only if it is perpendicular to P_i. Then the decomposition space is homeomorphic to E^3. A generalization of this theorem is proved which allows the nondegenerate elements of G to be arcs provided that there exist coverings of them by arbitrarily thin polyhedral cubes in a nice way (with respect to the planes P_i which intersect them). A generalization of a theorem due to R. H. Bing (Annals, 1957, Theorem 1, pp. 363-374) is also proved. (Received December 16, 1957.)

542-199. D. M. Mesner: An embedding theorem for 0, 1 matrices.

Necessary and sufficient conditions are found for the existence of an $m \times n$ matrix of 0's and 1's with (1) specified row totals $r_1,\ldots,r_m$ and column totals $s_1,\ldots,s_m$, (2) a given $m_0 \times n_0$ matrix of 0's and 1's as the submatrix determined by rows $1,\ldots,m_0$ and columns $1,\ldots,n_0$, where $m_0 \leq m$, $n_0 \leq n$. (Received December 16, 1957.)

542-200. Albert Nijenhuis: A generalization of Walker's differential operator.

An operator D linear over the real constants assigning to any (say, C^∞) vector field v a tensor field Dv, with rule for multiplication by functions $D(fv) = fDv + Df \otimes v$ (Df may be anything consistent with the formula) is uniquely extendable to all tensor fields in such a manner that D is linear over the constants and that the Leibnitz rule holds for tensor products and contractions. The Lie derivative of any tensor field T with respect to a vector field u, $T \to [u,T]$ is thus determined by $v \to [u,v]$ (Lie bracket). Walker [C. R. vol. 245, 1213 - 1215 (1957)] gave another example: if an almost complex structure J with nonvanishing torsion T is given, then $v \to W_T v = [v,\Gamma] + J\bar{\wedge}[Jv,T]$ also determines such an operator. This procedure is easily generalized to any vector form T of degree p satisfying $T\bar{\wedge}J = -pJ\bar{\wedge}T$. Practically equivalent is the operator W_T in an almost product structure $PP = P$, $Q = I - P$, with $PT = T$; $TP = 0$; and $W_T v = [Pv,T] + [Qv,P]\bar{\wedge}T$. An application to be studied

is e.g. the operator W_A associated with a family J_t of (a)most) complex structures; $A = J(dJ/dt)$, which actually is a covariant derivation. References for notation: Froelicher-Nijenhuis [Indag. Math. vol. 18(1956) pp. 338-359, 540-564; Proc. Nat. Acad. Sci. vol. 43 (1957) pp. 239-241]. (Received December 16, 1957.)

542-201. F. P. Peterson and Emery Thomas: Functional operations associated with nonadditive cohomology operations.

Two methods of defining functional cohomology operations have been given: the original definition by Steenrod (Ann. of Math. vol. 50 (1949)), and a recent definition by Peterson (Trans. Amer. Math. Soc. vol. 86 (1957)). In these definitions the functional operation has as value an element of a factor group of a cohomology group. In each case the subgroup which one factors by is obtained using the fact that the cohomology operations considered are additive (i.e., group homomorphisms). Our result is to show that the same form of subgroup is obtained (and hence, the same form of factor group) even if the primary cohomology operation is nonadditive. Two proofs are given, one for each definition. The second proof (based on Peterson's definition) introduces the concept of a principal fibre space, of which the space of loops of a space is a special case. An application of the result is made by computing an example of the functional operations associated with the generalized Pontrjagin operations; these are nonadditive cohomology operations defined by Thomas in Memoirs of the Amer. Math. Soc., vol. 27. (Received December 16, 1957.)

542-202. A. L. Shields: On complex difference equations.

Let μ be a complex measure with compact support in the plane. Let N be the set of entire functions f that are solutions of the difference equation $\int f(z + w)d\mu(w) \equiv 0$. Let the characteristic function $\phi(z) = \int \exp(zw)d\mu(w)$ have zeros a_n with multiplicities m_n. A. O. Gelfond has shown that one may associate with each $f \in N$ a canonical series of the form (1) $\sum P_n(z)\exp(a_n z)$, where P_n is a polynomial of degree $< m_n$, and that a subsequence of the partial sums converges to f uniformly on compact subsets. (This result was also obtained independently by H. S. Shapiro.) Theorem. If $f \in N$ and ϕ_1 is of exponential type with zeros b_n of multiplicities p_n and (2) $\sum Q_n(z)\exp(b_n z)$ has a sequence of partial sums converging to f uniformly on compact sets, then (2) is the canonical series for f. Here Q_n is a polynomial of degree $< p_n$. Corollary. If

μ_1, μ_2 are two such measures and if there is a greatest common divisor ϕ_3 for the characteristic functions ϕ_1 and ϕ_2 then $N_3 = N_1 \cap N_2$. This result was obtained by Gelfond in case ϕ_1 and ϕ_2 have only a finite number of common zeros (Received December 16, 1957.)

542-203. A. T. Butson: Sufficient conditions for an abstract group to be ordered. Preliminary report.

An R-group is one in which for every pair of elements x and y and every natural number n it follows from $nx = ny$ that $x = y$. It is well known that every ordered group is an R-group. In this paper the concepts of linear independence and basis are defined for an arbitrary group. Then it is shown that a locally-nilpotent R-group with a basis can be ordered. (Received December 17, 1957.)

542-204. C. C. Faith: On conjugates in division rings.

Let D be a noncommutative division ring with center C. If R is any subset of D, let R^* be the set of all nonzero elements in R, R' the centralizer of R in D, and $H(R) = \{x \in D^* | x^{-1}Rx = R\}$. If Δ is a division subring of D, $\Delta \neq D$, $\Delta \neq \Delta \cap C$, then the Cartan-Brauer-Hua theorem states that $H(\Delta) \neq D^*$ [Bull. Amer. Math. Soc. vol. 55 (1949) pp. 619-620]. The main result of this present note is the theorem: (A) $H(\Delta)$ cannot have finite index in D^*. Thus Δ has infinitely many distinct conjugates $x^{-1}\Delta x$, a result which contains Herstein's theorem [Bull. Amer. Math. Soc. vol. 7 (1956) pp. 1021-1022]. The method of Brauer is applied to obtain the following key lemma which seems new: Let $d \in \Delta$, $d \notin C$, and let $v \in D$, $v \notin \Delta$, such that $vd \neq dv$. Also let c_0, c_1, and c_2 be three distinct elements of $d' \cap \Delta$. Then $v + c_0$, $v + c_1$, and $v + c_2$ cannot all belong to the coset of $H(\Delta)$ determined by one of them. Thus when $d' \cap \Delta$ is infinite for some such d, (A) follows from the lemma. Otherwise, one may apply Jacobson's theorem on algebraic division algebras over a finite field [Ann. of Math. vol. 46 (1945) pp. 695-707], to reduce the problem to the case when Δ is a finite field having degree $n > 1$ over $\Delta \cap C$. (A) follows easily in this case. Although (A) implies the Wedderburn theorem on finite division rings, the present proof is not a new proof of this since Jacobson's theorem is used. (Received December 17, 1957.)

542-205. A. H. Copeland, Sr. and A. H. Copeland, Jr.: Convex geometry.

A convex geometry can be described roughly as a geometry which is based on the concept of convex hull (defined postulationally) and which occupies a position between Riemannian geometry and point-set topology. In terms of convex hull, one can define such concepts as geodesic, geometric independence, flat, simplex and limit point (called convex limit). The convex limit is used to define a topology. It is shown that, in general, an n-simplex is the homeomorph of a euclidean n-simplex, a finite dimensional flat is a manifold, and a finite dimensional compact convex element is the convex hull of its extreme points. The postulates are stated in terms of an atomic boolean ring B with unit 1. The unit is interpreted as a space and the atoms as points of the space. If 1 is a finite dimensional euclidean space, then one can associate with every element x in B its convex hull $\eta(x)$. However, for certain spaces (e.g. a sphere or torus) it may not be appropriate to associate a convex hull with every element, but only with those which are in some sense small. (Received December 20, 1957.)

542-206. C. C. Faith: Galois extensions in which every element with regular trace is a normal basis element. I.

Let K be a ring with identity, and let F be a simple ring (= simple ring with minimum condition throughout) such that K/F is a finitely generated right F-module. Then [Amer. Math. Soc. Colloquium Publications, vol. 37, 1956, p. 134], K/F has an independent right F- basis; (K:F) denotes the invariant basis number. If F is the fixring corresponding to a group G of automorphisms of K, then K/F is G-Galois. If also $G = (G_1,\ldots,G_n)$, and $(K:F) = n$, then, following [1] [Math. J. Okayama University vol. 7 (1957) pp. 77-81], K/F is strictly G-Galois. Then if K/F has an independent right F-basis of the form $u^{G_1}, \ldots, u^{G_n}$, $u \in K$, (α): K/F has a G-normal basis. For short, we say u is a G-NBE of K/F. If H is a subgroup of G, the H-trace $T_H(x)$ of $x \in K$ is the sum of the x^S, S ranging over H. Let Δ be a simple ring, $K \supseteq \Delta \supseteq F$, such that ($\beta$): K/$\Delta$ is strictly H-Galois, where H is normal in G. Then $T_H(u)$ is a P-NBE of Δ/F whenever u is a G-NBE of K/F, where P is the group of automorphisms of Δ/F induced by G. Our main result concerns the converse: (I) u is a G-NBE of K/F whenever $T_H(u)$ is a P-NBE of Δ/F. Theorem 1. Let (α) and (β) hold, and $K \neq \Delta$. Then (I) holds if and only if: (II) K has characteristic $\chi(K) = p$, and $(K:\Delta) = p^e$. This theorem generalizes the results (for

fields) announced by the author in [2] [Bull. Amer. Math. Soc. vol. 63 (1957) pp. 95-96]. (Received December 23, 1957.)

542-207. C. C. Faith: Galois extensions in which every element with regular trace is a normal basis element. II.

(The notation of the previous abstract will be assumed). Putting $\Delta = F$ in Theorem 1 we get: Theorem 2. If (α) holds, and $K \neq F$, then every $u \in K$ with regular G-trace is a G-NBE of K/F if and only if both: $\chi(K) = p$ and $(K:F) = p^e$. (The proof of this result in the case K is a division ring has been announced in [1]). Next conditions on K are obtained under which (α) holds for K/F. Results of T. Nakayama and F. Kasch are applied to the case where K and the centralizer of F in K are both simple to show that: (α) holds whenever K/F is strictly G-Galois. For the most part, the methods of the present note are the same as those of [2], namely, essential use is made of a G(F)-module isomorphism between G(F) and K, where G(F) denotes the group ring defined by G and F. It is shown that this isomorphism maps regular elements of G(F) onto G-NBE's of K/F. Then a structure theorem for G(F) is obtained which produces Theorem 1. Although it appears new, this theorem may be obtained in routine fashion from known results for G(F) in the case F is a field. First let $\tau = \sum_{i=1}^{h} H_i$, $h > 1$, where $H = (H_1,\ldots,H_h)$ is a normal subgroup of G; $B = \{\sigma \in G(F) \mid \sigma\tau = 0\}$ is an ideal of G(F). Then: (Theorem) B is contained in the radical of G(F) if and only if both: $\chi(F) = p$ and $h = p^e$. (The results of this present note will appear shortly in the Proc. Amer. Math. Soc.) (Received December 23, 1957.)

542-208. Stefan Bergman: Operators generating solutions of certain partial differential equations. I.

Let: $L_3(\psi) = \Delta_3\psi + F(Z,Z^*)\psi = 0$. Here $Z = (z + iy)/2$, $Z^* = (z - iy)/2$, $x = X$, $\Delta_3\psi = \psi_{xx} + \psi_{yy} + \psi_{zz} = \psi_{XX} + \psi_{ZZ^*}$ where x,y,z are the cartesian coordinates. F is an entire function of complex variables Z,Z^*. Let: $\gamma = 1 - \int_0^Z\int_0^{Z^*} F_1\delta_1 + \int_0^Z\int_0^{Z^*} F_1\int_0^{Z_1}\int_0^{Z_1^*} F_2\delta_2 - \ldots$, $\delta_n = \prod_{\mu=1}^{n} dZ_\mu\, dZ_\mu^*$, $F_n = F(Z_n,Z_n^*)$. Furthermore: $U^N \equiv [Z\zeta + X - Z^*\zeta^{-1}]^N = \sum_{k=0}^{2N} R^{(Nk)}\zeta^{N-k}$, $R^{(Nk)} = \sum_{\nu=0}^{k-[(k+1)/2]} C_{N,k-\nu}C_{k-\nu,\nu}Z^{N-k+\nu}(-Z^*)^\nu X^{k-2\nu}$ holds. Here $[k/2]$ denotes the largest integer which equals or is smaller than k/2. Expressions $\Gamma(Z,Z^*) = -\int_0^Z\int_0^{Z^*} T_1\delta_1 + \int_0^Z\int_0^{Z^*} F_1\int_0^{Z_1}\int_0^{Z_1^*} T_2\delta_2 - \ldots$, $T_n \equiv T(Z_n,Z_n^*)$, will be denoted as solutions of type A of $\psi_{ZZ^*} + F\psi + T = 0$. It is shown that the

functions $\Psi^{(N,k)}(X,Z,Z^*) = (\gamma/2\pi i)^{-1}\int_C U^N \zeta^{-(N+k)}\, d\zeta/\zeta$ $+\sum_{\nu=0}^{[k/2]} \Pi^{(N,k,\nu)}(Z,Z^*)X^\nu$ form a set of independent solutions of the equation $L_3(\Psi) = 0$. Here $\Pi^{(N,k,\nu)}(Z,Z^*)$ are solutions of type A of the system $N\gamma_{Z^*}P^{(N-1,k,k)} + \Pi^{(N,k,k)}_{ZZ^*} + F\Pi^{(N,k,k)} = 0$, $N\gamma_{Z^*}P^{(N-1,k,\nu)} - N\gamma_Z{}^{(N-1,k-2,\nu)}$ $+(N-k-2\nu+2)(N-k-2\nu+1)\Pi^{(N,k,\nu+2)} + \Pi^{(N,k,\nu)}_{ZZ^*} + F\Pi^{(N,k,\nu)}$ $= 0$, $\nu = k-2, k-4, \ldots, k-2[k/2]$. C denotes $[|\zeta| = 1]$. (Received January 16, 1958.)

542-209. Stefan Bergman: Operators generating solutions of certain partial differential equations. II.

Generalizing considerations of the previous abstract, the author defines integral operators $T_m(X,Z,Z^*;f) = \sum_{\kappa=0}^{[m/2]}[(m-2\kappa)!\kappa!]^{-1}f^{(N-\kappa)}(Z)X^{m-2\kappa}{}_{(-Z^*)}$ $+\sum_{\kappa=0}^{[m/2]} X^{m-2\kappa}\Gamma^{(m,m-2\kappa)}(Z,Z^*;f)$, $m = 0,1,2,\ldots$ transforming analytic functions $f(Z)$ of the complex variable Z, which are regular at the origin, into solutions of the equation $L_3(\psi) = \Delta_3\psi + F(Z,Z^*)\psi = 0$. Here $f^{(S)} = d^S f/dZ^S$ and $\Gamma^{(m,m-2\kappa)}(Z,Z^*;f)$ are conveniently chosen solutions of the system $[m!]^{-1}f^{(m+1)}(Z)\gamma_{Z^*} + \Gamma^{(m,m)}_{ZZ^*}(Z,Z^*;f) + F\Gamma^{(m,m)}(Z,Z^*;f) = 0$, $-[(m-2\kappa)!(\kappa-1)!]^{-1}f^{(m-\kappa)}(Z)\gamma_Z(-Z)^{\kappa-1} + [(m-2\kappa)!\kappa!]^{-1}f^{(m-\kappa+1)}(Z)\gamma_{Z^*}$ $\cdot(-Z^*)^\kappa + (m-2\kappa+2)(m-2\kappa+1)\Gamma^{(m,m-2\kappa+2)}(Z,Z^*;f) + \Gamma^{(m,m-2\kappa)}_{ZZ^*}(Z,Z^*;f)$ $+F\Gamma^{(m,m-2\kappa)}(Z,Z^*;f) = 0$, $\kappa = 1,2,\ldots,[m/2]$, $\Gamma_{ZZ^*} = \partial^2\Gamma/\partial Z\partial Z^*$. Writing $R_m(X,Z,Z^*;f) = 2^{-1}\gamma\sum_{k=0}^{[m/2]}[f^{(m-\kappa)}(Z)(-Z^*)^\kappa + \bar{f}^{(m-\kappa)}(Z^*)(-Z)^\kappa]X^{m-2\kappa}$ $\cdot[(m-2\kappa)!\kappa!]^{-1} + \sum_{\kappa=0}^{[m/2]}[\Gamma^{(m,m-2\kappa)}(Z,Z^*;f) + \bar{\Gamma}^{(m,m-2\kappa)}(Z^*,Z;f)]X^{m-2\kappa}$ one obtains solutions which for $Z^* = \bar{Z}$ are real. They have the property that $R_m(X,Z,0,f) = 2^{-1}[m!]^{-1}f^{(m)}(Z)X^m + \sum_{\kappa=0}^{[m/2]}[(m-2\kappa)!\kappa!]^{-1}f^{(m-\kappa)}(0)X^{m-2\kappa}{}_{(-Z)^\kappa}$. (Received January 16, 1958.)

542-210. T. J. McMinn: On the surface line segments of a convex set.

V. L. Klee (Bull. Amer. Math. Soc. Research Problem 63-6-5) raised these questions: Let B be the boundary of a convex subset C of Euclidean n-space, let S be the unit sphere $\{x:\|x\| = 1\}$, and let D be the set of directions (subset of S) of line segments in B. Is D of first category in S? Is (n-1)-dimensional measure of D zero? Both questions are answered affirmatively. Call a 1-facet a closed line segment in B not lying in the interior of a piece of a k-plane ($1 < k < n$) which lies in B. There is a countable collection of pairs of parallel (n-1)-planes such that every line segment of B intersects both members of at least one pair. We show that for any pair the set of directions of connecting lines other than

1-facets is of first category in S and of 1-facets is nowhere dense in S. Also both sets have zero (n-1)-measure. Proof for non-1-facets uses separability. Proof for 1-facets uses the fact that the set of closed middle thirds of 1-facets is disjoint allowing construction of a Lipschitz (and hence nonmeasure-increasing) function f on a closed subset of the intersection of B with one plane of the pair such that f(x) is the direction of the unique 1-facet through x. (Received January 20, 1958.)

542-211. S. G. Bourne and H. J. Zassenhaus: On the semiradical of a semiring.

The element r of the semiring S with commutative addition and strong zero is said to be left semi-regular if there exist elements r' and r'' satisfying $r + r' + r'' r = r'' + r' r$. As in S. Bourne, The Jacobson radical of a semiring Proc. Nat. Acad. Sci. U. S. A. vol. 37 (1951) pp. 163-170, a new Jacobson Radical R of S is obtained. The closure $\bar{1}$ of a left ideal 1 is defined as the set of all elements congruent 0 mod 1. A left ideal 1 is called semi-modular, if there is an element s not in $\overline{1 + R}$ such that $x \equiv xs(1)$ if $y + x \equiv xs(\bar{R})$ is solvable and if $1 = \bar{1}$. The intersection of all maximal semi-modular ideals is $\bar{R}$ and hence is called the semiradical of S. This is also the intersection of all semi-primitive ideals i.e. the annihilators of the factor modules to maximal semi-modular left ideals. The zeroid is the ideal of all b for which $b + x = x$ is solvable. If S satisfies the descending chain condition, the semiradical is nil-potent mod the zeroid. The radical defined by W. Slowikowski and W. Zawadowski, A generalization of the maximal ideals method of Stone and Gelfand, Fund. Math. vol. 42 (1955) pp. 215-231, is a special case of the semiradical defined above, which answers the Math. Reviewers question. By dropping the semi-prefix, in the ring case one obtains standard notions of ring theory. (Received January 24, 1958.)

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543-14. Edward Halpern: Twisted polynomial hyperalgebras.

Let J be an integral domain with characteristic p. A monogenic twisted polynomial J-algebra is the free J-module generated by a sequence $x_0, x_1, \ldots, x_{n-1}$ $(2 < h \leqq \infty)$ with multiplication given by: $x_m x_n = 0$ for $m + n \geq h$, $x_m x_n = t_{m,n} x_{m+n}$ for $m + n < h$ where $t_{m,n} \in J$ and satisfy $t_{m,n} \neq 0$, $t_{m,0} = 1$, $t_{0,n} = 1$, $t_{m,n} = t_{n,m} t_{m,n} t_{m+n,k} = t_{m,n+k} t_{n,k}$. It is denoted $J[x,h,t]$; $x = x_1$, h is its height, t is its type. Let H be a torsion-free graded J-algebra. If $x \in H$ has positive degree then the integral closure of the subalgebra generated by x is a graded subalgebra $J[x,h(x), t(x)]$. If each $t_{m,n}(x)$ differs from the binomial coefficient (m,n) by a unit factor then x is of binomial type. A special pseudogenerating system is a subset of homogeneous elements X such that (i) H is the integral closure of the subalgebra spanned by X, (ii) the inclusion map $X \to H$ defines an algebra monomorphism $i: \otimes_{x \in X} J[x,h(x), t(x)] \to H$. If i is onto then H is a twisted polynomial algebra denoted by $J[X,h,t]$. Theorem: Let H be a torsion-free Hopf algebra over J with $p = 0$. If X is a special pseudogenerating system such that (a) each x is primitive, (b) each x is of binomial type, (c) the submodule spanned by X is a direct summand, then $H = J[X,h,t]$. If condition (c) is slightly modified, the result holds also if $p \neq 0$. (Received December 6, 1957.)

543-15. J. W. Andrushkiw: A note on monotonely decreasing functions.

Let f(x) be a function of real variable x which is integrable, positive, and monotonely decreasing to 0 in the interval $(0, \infty)$ (i.e. $\lim f(x) = 0$ as $x \to \infty$). Further, denote $a_n(\ell,m) = \sum_{k=\ell n}^{mn} f(k) - \int_{\ell n}^{mn} f(x)dx$ where $0 \leqq \ell < m$ are integers. It follows that: 1. $0 < f(mn) < a_n(\ell,m) < f(\ell n) < f(0)$, $a_{n+1}(\ell,m) < a_n(\ell,m)$. 2. $\lim_{n \to \infty} a_n(0,m) = C$, $0 \leqq C < f(0)$. 3. $\lim_{n \to \infty} a_n(\ell,m) = 0$ if $\ell > 0$. 3'. Corollary: If $\lim_{n \to \infty} \int_{\ell n}^{mn} f(x)dx = L$ exists, it is also $\lim_{n \to \infty} \sum_{k=\ell n}^{mn} f(k) = L$ (e.g. $\lim_{n \to \infty} \sum_{k=\ell n}^{mn} \operatorname{arc\,cot} k = \log(m/\ell)$). (Received December 23, 1957.)

543-16. Norman Meyers: Expansion of solutions of second order linear elliptic equations about infinity.

The results of this paper hold for the general linear elliptic equation $a_{k\ell}D_{k\ell}U + a_kD_kU + aU = f$ in all dimensions but for simplicity they are stated here only for equations $a_{k\ell}D_{k\ell}U = 0$ in more than two dimensions. Let x be point in some neighborhood of infinity. Let $r = |x|$ and $\omega = x/|x|$. The following proposition is proved. Assume $a_{k\ell}(x) = \delta_{k\ell} + \sum_{\lambda=1}^{N} a_{k\ell}^{(\lambda)}(\omega)r^{-\lambda} + A_{k\ell}^{(N)}$ where $a_{k\ell} \in C^{i+\alpha}$, $A_{k\ell}^{(N)} = O_i(r^{-N-\lambda})$ $(0 < \epsilon < 1)$ and $D^iA_{r\ell}^{(N)}$ satisfies a certain α-Hölder condition in the neighborhood of infinity. If $U = o(r^{1+p})$, p being the smallest such integer, then $U(x) = H_p + \sum_{\lambda=1}^{N} r^{p-\lambda} \sum_{\nu=0}^{\lambda} U_{\lambda\nu}(\omega)(\log r)^{\nu} + U^{(N)}$ with the condition $U_{\lambda\nu} = 0$ when $\nu > \max(p,0) + \max(\lambda - p - n + 3, 0)$. H_p is a spherical harmonic of degree p; $U_{\lambda\nu} \in C^{i+2+\alpha}$; $U^{(N)} = O_{i+2}(r^{p-N-\epsilon})$ and $D^{i+2}U^{(N)}$ satisfies a certain α-Hölder condition. It is shown separately that if $U = O(r)$ then $U = \text{const.} + O(r^{2-n})$. This statement together with the above essentially constitute the expansion theorem. Similar results hold in the general case. The proof employs Schauder estimates coupled with Green's representation of U which involves the fundamental solution of Laplace's equation. The method can be used for equation of higher order once certain properties of fundamental solutions of equations with constant coefficients have been established. This has not yet been done. (Received December 23, 1957.)

543-17. W. E. Roth: Determinants of certain composite matrices.

The principal result established in this paper is the following: If the matrix A has elements a_{ij}, $i, j = 1,2,\ldots, n$, in the field F and if its characteristic function is $a_0(x^2) - xa_1(x^2)$, then the determinants of $a_0(A^2)$ and $a_1(A^2)$ are $\pm a_0(0)S_{n-1}^2\{a_0(y), a_1(y)\}$ and $\pm S_{n-1}^2\{a_0(y), a_1(y)\}$ respectively, where $S_{\beta+\gamma}^2\{b(x), c(x)\}$ is the square of the Sylvester eliminant for the polynomials b(x) and c(x) of degree β and γ. The proof rests upon some observations on the determinants of composite matrices (A_{ij}), $i, j = 1,2,\ldots, r$, whose elements are commutative $n \times n$ matrices. (Received December 26, 1957.)

543-18. W. E. Roth: The matric equation $X^2 - Y^2 = A$.

The equation $X^2 - Y^2 = A$, where A is an $n \times n$ matrix with elements in the field F, has an infinitude of solutions of the form $X = M + N$, $Y = M - N$, where M is nonsingular and such that its characteristic function, $m_0(x^2) - xm_1(x^2)$, is such that $m_0(x^2)$ and $m_1(x^2)$ are relatively prime in the polynomial domain F(x)

of F. And each such choice of M leads to a unique solution X, Y of the given equation. (Received December 26, 1957.)

543-19. Jacob Wolfowitz: The maximum achievable length of an error correcting code.

The problem of transmission of messages through a noisy channel with a finite memory, as described in the author's paper, The coding of messages subject to chance errors, Illinois Journal of Math., 1957, is considered. A number J is defined by certain algebraic and analytic operations on the channel probability function which are too long to describe here. It is proved that, for any positive ϵ and n sufficiently large, there exists a code of length $2^{n(J-\epsilon)}$, and there does not exist a code of length greater than $2^{n(J+\epsilon)}$. Roughly speaking, therefore, 2^{nJ} is the maximum achievable length of code. (Received December 27, 1957.)

543-20. Walter Rudin: On the structure of maximum modulus algebras.

Let K be the closed unit disc in the complex plane, U the interior of K, C = K - U. An algebra A of continuous complex functions on K is called a maximum modulus algebra if to every f in A there is a point z_0 on C such that $|f(z)| \leq |f(z_0)|$ for all z in K. It is known [Duke Math. J. vol. 20 (1953) pp. 449-458] that if A is a maximum modulus algebra such that (1) A contains a non-constant function which is analytic in U, (2) A contains a function g which is one-to-one on K, then every member of A is analytic in U; in this case, A is generated by g. Later [Proc. Amer. Math. Soc. vol. 7 (1956) pp. 808-811] it was shown that the conclusion becomes false if (2) is omitted from the hypotheses; the algebra in this example had two generators and did not separate points on K. In the present paper it is shown that there exist maximum modulus algebras (finitely generated) which satisfy (1), which separate points on K, and which contain functions which are not analytic in U. (Received December 30, 1957.)

543-21. Eckford Cohen: Representations of even functions (mod r). III. Special topics.

This paper deals with miscellaneous topics concerning even arithmetical functions (mod r). (1) Inequalities involving averages of even functions f(n,r) are derived, using elementary results of probability theory. These are illustrated with a number of special functions. (2) Approximations to the mean

values of functions f(n,r) are obtained on the basis of a trigonometric estimate, the latter being an improvement of an earlier estimate of Carmichael. (3) Denoting by J_r the ring of residue classes (mod r), simple expressions are obtained for the number of solutions $G_s(\rho)$ of $\rho = \pi_1 + \ldots + \pi_s$ where $\rho \in J_r$ and the π_i are primes of J_r. These formulas are obtained by interpreting $G_s(\rho)$ as an even function (mod r). (4) Finally, a new proof is obtained for the orthogonality of the function $c_k(n,r)$, discussed by the author in an earlier paper (Duke Math. J. vol. 22 (1955) pp. 543-550). (Received January 6, 1958.)

543-22. I. N. Herstein: On a question by Kaplansky in ring theory.

In the proceedings of the algebra conference held at Shelter Island, Kaplansky raised the following question: if the group ring of a group G over a field K has the property that every element can be written as a scalar plus a nilpotent then is G a locally finite group all of whose elements have order a power of p, where the prime p is the characteristic of K? In this note this is proved except for the local finiteness of G. (Received January 9, 1958.)

543-23. Silvio Aurora: Multiplicative norms for metric rings. I.

In this abstract, all rings have a unit element, e. A real-valued function N on a ring R is called a norm for R when $N(x) = 0$ if and only if $x = 0$, $N(x - y) \leqq N(x) + N(y)$ for all x and y, and $N(xy) \leqq N(x)N(y)$ for all x and y. If the norm N is such that $N(xy) = N(x)N(y)$ for all x and y, then N is called an absolute value. A ring R, together with a norm (absolute value) for R, is called a metric ring (ring with absolute value). For any ring R, the sets G and S, and the notions of regular elements and singular elements are introduced as in C. E. Rickart, The singular elements of a Banach algebra, Duke Math. J. vol. 14 (1947) pp. 1063-1077. A metric ring is called a Q-ring if the group G of regular elements is open; for instance, any complete metric ring is a Q-ring. The function $\ell(a) =$ glb of $N(ax)/N(x)$ over all nonzero x is introduced; an element a is called a generalized left-divisor of zero if and only if $\ell(a) = 0$. The set of all generalized left-divisors of zero is denoted by Z^ℓ. Similarly, Z^r is introduced, and also $Z = Z^\ell \cup Z^r$, the set of generalized divisors of zero. H is the complement of Z. For a metric Q-ring R, the boundary of G is contained in $Z^\ell \cap Z^r$, so if the complement of 0 is connected, then either R is a division ring or it contains a nonzero generalized divisor of zero. This generalizes a result of Šilov (On the extension of maximal ideals, C. R. (Doklady) Acad. Sci. URSS. vol. 29 (1940) pp. 83-84). (Received January 9, 1958.)

543-24. Silvio Aurora: Multiplicative norms for metric rings. II.

In the sequel it is assumed that $N(e) = 1$. For a metric ring R, the sets $\mathscr{L}(R) = \{a \mid N(ax) = N(a)N(x)$ for all x in R$\}$ and $\mathscr{G} = \{x \mid x$ is regular and $N(x)N(x^{-1}) = 1\}$ are introduced to measure the extent to which the norm is multiplicative, that is, resembles an absolute value. Various successively weaker hypotheses on the multiplicative character of the norm of a metric Q-ring are considered: M1. The norm is an absolute value; M2. $\mathscr{G} = G$; M3. $\mathscr{G}$ is an open set; M4. $\mathscr{G}$ fails to be nowhere dense; M5. $\mathscr{L}(R)$ fails to be nowhere dense. The last three hypotheses are in fact equivalent for any metric Q-ring, and other relations among the five hold under various topological assumptions. A metric Q-ring in which S is nowhere dense and for which M1 or M2 holds is necessarily a division ring with absolute value. A metric Q-ring in which H is connected and for which M1 holds is necessarily a division ring. For a complete metric ring R in which the complement of 0 is connected, if any one of M1-M5 holds, then R is algebraically and topologically isomorphic to the complexes, the quaternions, or the field of order 2. For a complete metric ring R in which S is of first category and G is connected, if any one of M1-M5 holds, then R is algebraically and topologically isomorphic to the complexes or the quaternions. (Received January 9, 1958.)

543-25. Silvio Aurora: Multiplicative norms for metric rings. III.

By applying the previous results to normed algebras over the real field, generalizations are obtained of familiar results of S. Mazur (Sur les anneaux linéaires, C. R. Acad. Sci. Paris vol. 207 (1938) pp. 1025-1027) and R. E. Edwards (Multiplicative norms on Banach algebras, Proc. Cambridge Philos. Soc. vol. 47 (1951) pp. 473-474). In particular, a real normed algebra in which M1, M3, M4 or M5 holds is algebraically isomorphic and isometric to the reals, complexes, or quaternions; for a real normed algebra which is a Q-ring, M2 implies the same conclusion. By a μ-group of a metric ring is meant a multiplicative group such that $N(xy) = N(x)N(y)$ for all x and y in the group. For a real normed algebra distinct from the reals, the complexes, and the quaternions, every μ-group is nowhere dense. (Received January 9, 1958.)

543-26. Silvio Aurora: On power multiplicative norms. I.

A real-valued function N on a ring R is called a pseudonorm if $N(0) = 0$, $N(x - y) \leqq N(x) + N(y)$ and $N(xy) \leqq N(x)N(y)$ for all x and y; N is called a norm

if, in addition, $N(x) = 0$ only when $x = 0$. A pseudonorm (norm) N such that $N(xy) = N(x)N(y)$ for all x and y is called a pseudo absolute value (absolute value). A pseudonorm N is said to be stable if $N(xy) = N(yx)$ for all x and y; by a power multiplicative pseudonorm is meant a pseudonorm N such that $N(x^2) = N(x)^2$ for all x. A family of pseudonorms on a ring is said to be bounded if for every x the set of values assumed at x by members of the family is bounded. For a bounded family, the join of the family is defined to be the function which takes as value at x the least upper bound of the set of values taken at x by members of the family; the join of a bounded family of pseudonorms is also a pseudonorm. The fundamental result of this paper asserts that every stable power multiplicative pseudonorm on a ring R is the join of a bounded family of pseudo absolute values on R. Corollary: A stable power multiplicative norm on a simple ring is the join of a bounded family of absolute values on that ring. (Received January 9, 1958.)

543-27. Silvio Aurora: On power multiplicative norms. II.

In this abstract, all rings are assumed to have a unit, e. A pseudonorm N is said to be non-archimedean if $N(x + y) \leqq \max(N(x),N(y))$ for all x and y, and archimedean otherwise. An archimedean pseudonorm which is the join of a bounded family of archimedean pseudo absolute values is said to be strictly archimedean. A division ring R which admits a stable archimedean power multiplicative norm is algebraically isomorphic to a division subring of the quaternions; if the norm is strictly archimedean and nondiscrete, then R is algebraically and topologically isomorphic to a division subring of the quaternions. In particular, a connected division ring with a stable power multiplicative norm is necessarily strictly archimedean and hence algebraically and topologically isomorphic to a division subring of the quaternions. Thus, a complete, connected division ring with a stable power multiplicative norm is algebraically and topologically isomorphic to the reals, complexes, or quaternions. A description of all power multiplicative pseudonorms for the ring of integers and for the field of rationals are also given in this paper. (Received January 9, 1958.)

543-28. Silvio Aurora: On certain regular metric rings.

Kaplansky has shown (Regular Banach algebras, J. Indian Math. Soc. vol. 12 (1948) pp. 57-62) that every regular Banach algebra is finite-dimensional. Using similar methods, it is shown that a regular, complete metric ring in which

all idempotents commute and in which 0 is not interior to the set of left zero-divisors must be the direct sum of finitely many division rings. A regular, connected, complete metric ring is the direct sum of finitely many full matrix rings over connected, complete metric division rings. The results of the preceding abstract can then be employed to show that a regular, connected, complete metric ring with stable power multiplicative norm is the direct sum of finitely many division rings, each algebraically and topologically isomorphic to the reals, the complexes, or the quaternions; such a ring is therefore algebraically and topologically isomorphic to a finite-dimensional Banach algebra. (Received January 9, 1958.)

543-29. H. W. Oliver: Spectra in a real Banach algebra.

In the Gelfand representation theory of commutative Banach algebras, with complex scalars, a crucial step is the proof that the spectrum of any element is nonvoid. This proof is usually carried out by an application of analytic function theory. For an algebra with real scalars, the analogous result is obtained by imbedding the real algebra in a complex algebra, and applying the known results. This paper, using only real variable methods, and without imbedding in a complex algebra, obtains proofs for various known results, of which the following are samples: (i) Let A be a real Banach algebra with unit e. Let x be any element of A. Define the quadratic spectrum of x, $\sigma_q(x)$, to be $\{(\xi,\eta);\ \xi,\eta \text{ real}, (x-\xi e)^2+\eta^2 e \text{ is not invertible}\}$. Then $\sigma_q(x)$ is not empty (ii) If A in (i) is also a field, A is algebraically isomorphic to the field of complex numbers or the field of real numbers. (iii) If A in (i) has the additional property that $x^2+e=0$ has no solution x in A, then the (ordinary) spectrum of any element is not empty. The proof of (i) uses the topology of the plane (via the theory of harmonic functions); but the proof of (iii) uses only the topology of the real line. (Received January 10, 1958.)

543-30. Walter Rudin: Fourier-Stieltjes transforms and weak almost periodic functions.

Let G be a locally compact abelian group. The question considered in this paper is whether every weak almost periodic function (in the sense of Eberlein, Trans. Amer. Math. Soc. vol. 67 (1949)) lies in the uniform closure of the set of Fourier-Stieltjes transforms on G. It is shown that the answer is negative if G is the integer group and if G is the real line. The proof depends on certain lacunarity properties of trigonometric series. (Received January 10, 1958.)

544-45. M. P. Shützenberger; Random walk on monoids.

Let P be a submonoid of the free monoid A generated by the finite set $\{a_i\}$. Assume that P satisfies $\mathcal{U}_r$, $\mathcal{U}_\ell$, $\mathcal{N}$ and $\psi_P A$, finite (notations of Bull. Amer. Math. Soc. Abstracts 63-2-187). If the a_i's are produced independently and with fixed probabilities $p_i \neq 0$ (Cf. H. Kesten, Bull. Amer. Math. Soc. Abstract 63-6-637), it can be proved that, as in the special instance of groups, the average recurrence time of the event [$a \in P$] (where $a \in A$) does not depend on the p_i's and is equal to the index of the Suschkevitsch group of $\psi_P P$ in that $\psi_P A$. This phenomena would not happen if the condition ($\mathcal{U}_r$ and $\mathcal{U}_\ell$) was replaced by ($\mathcal{U}_r$). (Received February 12, 1958.)

544-46. M. P. Schützenberger: Homomorphisms of a monoid onto a group.

The n.a.s.c. given by P. Dubreil, F. W. Levi and R. R. Stoll (ref. in R. R. Stoll. Amer. J. Math. vol. 73 (1951) pp. 475-481) for $Q \subset A$ with $Q^2 \subset Q$ to be the kernel of the homomorphism ψ_Q of A onto the union of a group and a zero can be improved as follows: Q ($\supset Q^2$) satisfies $\mathcal{U}, \mathcal{U}_i'$(if $aQb \cap Q \neq \emptyset$, then $aQb \subset Q$) and $\mathcal{N}_0'$(if $AaA \cap Q \neq \emptyset$ then, either $aA \cap Q \neq \emptyset$ or, $Aa \cap Q \neq \emptyset$). This last condition can be dispensed with if and only if $\psi_Q A$ admits minimal right (or left) ideals distinct from its zero. (Received February 12, 1958.)

544-47. D. W. Fox: Uniqueness theorems for singular Cauchy problems. Preliminary report.

Consider the singular Cauchy problem for the equation $L(u) = u_{tt} + (k/t)u_t - A(u) = 0$ in which k is a real constant and A is an analytic elliptic operator in m variables which does not depend on t. (See Bureau, Communications on Pure and Applied Mathematics vol. 8 (1955) pp. 143-202). The following theorems are proved: I. If $u \in c^2$ and $L(u) = 0$ with $k \neq 0$ for $t > 0$ and $u \in c'$ for $t \geq 0$ then $u_t(x, 0) = 0$. II. If $u \in c^2$ and $L(u) = 0$ with $0 \leq k < 1$ for $t > 0$, and $u \in c^0$ for $t \geq 0$ and the limits, $\lim_{t \to 0^+} u_{x_i}(x,t)$, $\lim_{t \to 0^+} t^k u_t$ converge uniformly with respect to x in each bounded set and the last has the value zero then $\lim_{t \to 0^+} \int u_t^2 ds = 0$ for each x_0, t_0 ($t_0 > 0$) where the integration is taken over the intersection of the reverse characteristic conoid which has

its vertex at x_0,t_0 with the hyperplane $t = \tau$. III. If $u \in c^2$ and $L(u) = 0$ with $k \geqq 0$ for $t > 0$ and $u \in c^0$ for $t \geqq 0$ and if $u(x,0) = 0$, $\lim_{t \to 0^+} u_{x_i} = 0$ and $\lim_{\tau \to 0^+} \int u_t^2 ds = 0$ (the limits being as in II) then $u \equiv 0$. In the case $A(u) = \Delta u + c(x)u$ with $c(x)$ continuous II is valid for $k \geqq 0$. The proofs of these assertions are based on techniques of Zaremba (Rendiconti Accademia Dei Lincei vol. 24 (1915) pp. 904-908), Friedrichs and Lewy (Math. Ann. vol. 98 (1928) pp. 192-204) and Walter (Math. Zeit. vol. 67 (1957) pp. 361-376). (Received February 13, 1958.)

544-48. John DeCicco: The group of bilinear transformations of a quadratic extension Γ of a field K.

Any nondegenerate quadric Q in a projective space S_3 of three dimensions based on a ground field K, not of characteristic two, can be defined in nonhomogeneous coordinates (x,y,w), by the canonical quadratic equation: $w = x^2 + \beta xy - \alpha y^2$. If $z = x + iy$ where $i^2 = \alpha + \beta i$ is a binary element of a quadratic extension Γ of K, the equation of Q is given by the norm N(z) of $z = x + iy$. Thus $w = N(z) = z\bar{z}$. The bilinear transformations T defined over Γ are of the form $Z = (az + b)/(cz + d)$, where $a d - b c$, is neither zero nor a proper divisor of zero of the quadratic ring Γ. These form a six-parameter group G_6. If Q is not a cone, this G_6 and the products of any such T by the reflection $Z = \bar{Z}$, form the total projective group G_6^* of Q. If Q is a cone, the transformations of G_6^* and the products of such correspondences by the transformation $X = x$, $Y = my$, where $m \neq 0$, form the total projective group G_7^* of the quadric cone Q. By means of the binary variable $z = x + iy$, the projective geometry of any given quadric surface Q may readily be developed. (Received February 26, 1958.)

544-49. Oswald Wyler: Ideal points and order axioms in an incidence geometry.

The author has given axioms of incidence and of order in two previous papers (Incidence geometry, Duke Math. J. vol. 20 (1953) pp. 601-610; Order in projective and in descriptive geometry, Compositio Math. vol. 11 (1953) pp. 60-70). In the present paper, the projective geometry of ideal points is considered for a geometry satisfying both axiom systems referred to above. It is shown that the order relation for actual points and lines can be extended to ideal points and lines in such a way that the usual axioms of order for a projective geometry are satisfied. (Received February 27, 1958.)

544-50. K. I. Appel: Horn sentences in identity theory.

Horn has shown (A. Horn, J. Symb. Logic, vol. 16, p. 14) that any property expressible by a Horn sentence is preserved under direct product. In the present paper, it is proved that any property preserved under direct product, and expressible by a sentence of the predicate calculus with equality as the only predicate, is expressible by a Horn sentence of the form: $\forall x_1 \ldots x_n \exists w_1 \ldots w_k \wedge(\wedge(x_i = w_j) \supset (x_s = x_t))$. The properties all concern cardinalities of models and the construction of the sentences utilizes properties of the partitions of a finite set. (Received March 3, 1958.)

544-51. D. F. Dawson: Concerning a property of continued fractions.

"The continued fraction F, with sequence of approximants $\{F_p\}$, has property A" means there exists a point v such that each of the sequences $\{F_{2p-1}\}$ and $\{F_{2p}\}$ contains an infinite subsequence convergent to v. For each complex number sequence a, let f(a) denote the continued fraction $1/\sqrt{1 + a_1}/\sqrt{1 + a_2}/\sqrt{1 + \ldots}$. If $b_1 = 1$, $a_p \neq 0$, $1/b_{p+1} = a_p b_p$, p = 1,2,3,..., then for f(a) to have property A it is necessary that the series $\sum|b_p|$ diverge. Among sufficient conditions are the following: (1) the odd (even) part of f(a) converges absolutely and the series $\sum|a_1a_3\ldots a_{2p-1}||a_2a_4\ldots a_{2p}|^{-1}$ ($\sum|a_2a_4\ldots a_{2p}||a_1a_3\ldots a_{2p+1}|^{-1}$) diverges, (2) f(a) is partially bounded, the odd (even) part of f(a) converges, and there exists k such that $B_{2p} \neq 0$ ($B_{2p+1} \neq 0$) if $p > k$, where B_q is the denominator of the qth approximant of f(a), (3) the odd (even) part of f(a) converges and there exists M such that $|a_p| < M$, p = 1,2,3,..., (4) $r_{2p}|1 + a_{2p-1} + a_{2p}| \geq r_{2p}r_{2p-2}|a_{2p-1}| + |a_{2p}|$, $r_0 > 0$, p = 1,2,3,..., and the series $\sum|a_1a_3\ldots a_{2p-1}||a_2a_4\ldots a_{2p}|^{-1}$ diverges. For each complex number sequence b, let g(b) denote the continued fraction $1/\sqrt{b_1 + 1}/\sqrt{b_2 + 1}/\sqrt{b_3 + \ldots}$, with sequence of approximants $\{g_p\}$. A more comprehensive result than the sufficiency of (1) is that if $\{g_{2p-1}\}$ ($\{g_{2p}\}$) converges absolutely and the series $\sum|b_{2p-1}|$ ($\sum|b_{2p}|$) diverges, then g(b) has property A. (Received March 3, 1958.)

544-52. Edward Fadell: An extension of a theorem of A. Dold and R. Thom.

Consider the category of pairs (G,G'), where G is a 0-connected commutative topological group and G' is a 0-connected closed subgroup. Let $C_n(G)$ denote the set of maps T: $\Delta_n \rightarrow G$, where Δ_n is the fundamental n-simplex. The operation in G induces an operation in $C_n(G)$ and hence $C_n(G)$ is an abelian

group. Define ∂: $C_n(G) \to C_{n-1}(G)$ by setting $\partial T = \sum(-)^i T^i$, where T^i are the faces of T. Then $\partial\partial = 0$ and $C(G) = \{C_n(G), \partial\}$ is a chain complex. Set $C(G,G') = C(G)/C(G')$. Theorem. The homology groups of C(G,G') are isomorphic to the homotopy groups of G mod G'. Now, let (X,A) denote a polyhedral pair and G an abelian (discrete) group. Let GX denote the reduced singular 0-cycles of X with coefficients in G. Then, using the metric on X, it is possible to topologize GX so that it becomes a regular commutative topological group which is 0-connected. Since A is closed in X, GA turns out to be closed in GX. Theorem. The usual homology groups (reduced) H(X,A;G) are isomorphic to the homology groups of C(GX,GA) as defined above. Hence it follows that with each polyhedral pair (X,A) and coefficient group G there is associated a pair of commutative topological groups (GX,GA) such that for $n \geqq 0$, $H_n(X,A;G)$ and $\pi_n(GX,GA)$ are isomorphic. This result extends a result of Dold and Thom (C. R. Acad. Sci. Paris vol. 242 (1956) pp. 1680-1682). (Received March 3, 1958.)

544-53. M. A. Martino: An extension of a spectral theorem of M. A. Naĭmark.

Theorem: Let A be a closed linear transformation of a Banach space B into itself and k a positive integer such that the domain of A^k is dense in B. Assume the resolvent $R_\lambda = (\lambda I - A)^{-1}$ is completely continuous for some $\lambda = \hat{\lambda}$ and that for each element x belonging to a dense set $S \subset B$ there exists a sequence of concentric circumferences $\{C_n\}$ in the complex plane satisfying: (1) each C_n belongs to the resolvent set of A; (2) the radius r_n of C_n tends to infinity with n; (3) $\lim_{n\to\infty} \text{l.u.b.}_{\mu\in C_n} \|R_\mu x\| \cdot r_n^{1-k} = 0$. Then the eigenvectors and generalized eigenvectors (collectively called "canonical" vectors below) span B and all members of $R_\mu^k(S)$ admit a norm-convergent expansion in canonical vectors. (Naĭmark's result requires $k = 1$.) Proof: Order the eigenvalues of A so that $\lambda_1, \lambda_2, \ldots, \lambda_{k_n}$ are those interior to C_n. Then if $\hat{\lambda}$ is interior to C_n we have $(1/2\pi i)\oint_{C_n} (R_\mu x)(\hat{\lambda} - \mu)^{-k} d\mu = \sum_{j=1}^{k_n} G_j(\hat{\lambda})x - R_{\hat{\lambda}}^k x$, where $G_j(\hat{\lambda})$ is the residue of the integral at λ_j. Since (3) implies the contour integral vanishes as n tends to infinity, (4) $R_{\hat{\lambda}}^k x = \lim_{n\to\infty} \sum_{j=1}^{n} G_j(\hat{\lambda})x$. But $G_j(\hat{\lambda})x$ is a finite linear combination of canonical vectors, so (4) implies that the canonical vectors span $R_{\hat{\lambda}}^k(S)$, which is dense in B. (Received March 3, 1958.)

544-54. P. F. McDougle: Mapping and space relations.

For notation and terminology see Abstract 538-32. X is an M space pro-

vided each sequence in X converges to at most one point. X is an E space provided every set in X has sequences converging to each of its limit points. If y is interior to f(U) for each y in Y and neighborhood U of $f^{-1}(y)$, then f is P_1. If $f^{-1}(C)$ is compact for each compact set C in Y then f is compact. The following are proved: (1) If X is ME and f is quasi-compact then Y is M if and only if f is semi-closed, (2) If X is ME and f is quasi-compact then Y is ME if and only if f is semi-closed and P_1, (3) If X is metric and f is closed or compact then Y is metric if and only if f is P_2. For decompositions of an ME space X there is shown an equivalency between a semi-closed natural mapping and a canonical [Martin: DMJ (1954) pp. 463-469] decomposition, and between a P_1 natural mapping and a quasi-continuous [Malbon: University of Virginia Dissertation (1955)] decomposition. (Received March 3, 1958.)

544-55. Paul McDougle: Semi-compact mappings.

See preceding abstract for terminology. Let f(X) = Y be continuous where X,Y are topological spaces, X being ME. Then f is semi-compact provided whenever $C \subset X$ is compact and (x_i) is a sequence in $f^{-1}(C)$ with $f(x_i)$ distinct, then (x_i) has a limit point in X. The following are proved: (1) f is closed if and only if f is quasi-compact, semi-compact, and semi-closed, (2) If Y is M then f generates an u.s.c. decomposition if and only if f is semi-compact, (3) If f is quasi-compact then f is compact if and only if f is closed and has compact point inverses, (4) If X is locally compact and is decomposed into connected sets with bicompact boundaries, the following are equivalent (a) the decomposition in u.s.c. (b) the hyperspace X' is M (c) X' is locally compact ME, (5) In (4), "ME" can be repaced by "separable metric" in X and X'. The results of (4) and (5) may be applied to: (6) If Y is T_1 there is a unique factorization f = Lm where m is quasi-compact with connected point inverses and L is light, (7) In (6) if f has compact point inverses then (a) m is monotone (b) if m is P_1 then L is open if and only if f is quasi-open. (Received March 3, 1958.)

544-56. E. C. Posner: Two theorems on (C,1) summability.

THEOREM A: Let T be a regular summation process which sums every (C,1) summable sequence. (In particular, every auxiliary mean must be defined for every (C,1) summable sequence.) Then T includes (C,1), i.e., the sums are the same. To prove this, first prove that the sums coincide on the bounded (C,1) summable sequences, which form a dense linear manifold in the Banach

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space of (C,1) summable sequences, where the norm is the supremum of the auxiliary means. Then show that if (t_{mn}) is the matrix for T, $t_{mn} = o_m(1/n)$. Then apply Theorem I of Divergent series, by G. H. Hardy, to prove that the T-sum is a continuous linear functional. Since it coincides with (C,1) sum, also continuous, on a dense set, the theorem is proved. THEOREM B: Let T be a regular summation process and f a function of a single complex variable such that, whenever $\{a_n\}$ is (C,1) summable, $\{f(a_n)\}$ is T-summable. Then there exist constants a,b such that $f(z) = az + b$ for all z. The proof resembles that of the first theorem. (Received March 3, 1958.)

544-57. N. A. Wiegmann: The structure of unitary and orthogonal quaternion matrices.

The following theorems are obtained: Every real quaternion unitary matrix P can be written in the form P = UDW where U and W are complex unitary matrices and D is a quaternion diagonal unitary matrix; conversely, every matrix of this form is a quaternion unitary matrix. Every orthogonal real quaternion matrix P can be written in the form $U(I \dotplus C)W$ where U and W are real orthogonal matrices, where I is an identity matrix, and where C is a direct sum of 2×2 matrices with nonzero quaternions of the form $q = a_1 i + a_2 j + a_3 ij$ along the diagonal, with real numbers b and -b in the off-diagonal positions, and with $q^2 + b^2 = 1$; conversely, every matrix of this form is an orthogonal quaternion matrix. (Received March 3, 1958.)

544-58. Anil Nerode: Some Stone spaces and recursion theory. I.

Let G be a set, let P(G) be its power set topologized by declaring $U(F) = [G' \subset G | F \subset G']$ open for finite $F \subset G$. Let $K \subset P(G)$ contain G, be closed under arbitrary meets and under unions of inclusion chains (inductive closure operation). Theorem. Suppose $C \subset P(G)$ closed, $X = C \cap K$ nonempty. (i) X is the Stone space of the sublattice of P(P(G)) generated by $U(F) \cap X$ for finite F. (ii) If k is the smallest member of K, a continuous $f: X \to X$ has $k \cup fk \cup f^2k \cup \ldots$ as smallest fixed point. (Applications. When X = P(G), (i) yields the results of Areskin (Mat. Sb. vol. 33 (1953) pp. 133-156) and Nerode (Abstract 63-6-647) on free distributive lattices. When X = C, X is precisely a property of finite character of subsets of G. All subalgebras of a finitary algebra G may be taken as K. If N is the natural numbers. and $C \subset P(N \times N)$ is all partial functions, (ii) is Kleene's first recursion theorem for partial recursive functionals

f.) If I is an ideal in a lattice L, let L/I denote L modulo the minimal congruence relation identifying I to a point. Theorem. Properties of finite character coincide with Stone spaces of L/I where L is a free distributive lattice with zero, unit, and ideal I. (Received March 4, 1958.)

544-59. P. J. Cohen: Factorization in group algebras.

Let G be a locally compact group, L(G) the group algebra of integrable functions. Then every function in L(G) can be written as the convolution of two others. In the case of locally Euclidean abelian groups this was proved by Rudin. The theorem is derived from a more general result concerning Banach algebras with approximate identities. Namely, if the algebra B has the property that there exists a constant C such that given x_i in B, $1 \leqq i \leqq n$ and $\epsilon > 0$, there exists an element e such that $\|e\| \leqq C$, and $\|ex_i - x_i\| < \epsilon$, then every element in B factors. Also if f is a non-negative function, it may not be the convolution of two non-negative functions. However, if f is strictly positive and continuous on a compact group, it is such a convolution. One cannot extend this to continuous functions which are only non-negative. (Received March 5, 1958.)

544-60. W. E. Deskins: On endomorphisms of nonabelian groups.

Under the usual operations of iteration and element-wise addition, a set of endomorphisms of a nonabelian group G generates a system which will be called a distributively-generated near-ring (a DGN-ring). A structure theory is developed for DGN-rings with the DCC on right modules and simple DGN-rings are considered. A simple DGN-ring R can be decomposed into components $e_iRe_j, e_iRe_i, e_jRe_j$ idempotents, and any element of R is expressible as a sum of component elements. Theorem. If these component expressions are unique, then R is isomorphic with a total matric algebra over a nearfield which is isomorphic with e_iRe_i. If, furthermore, G is a finite group, then e_iRe_i is a field. Other results of a similar nature are obtained. (Received March 5, 1958.)

544-61. Morris Marden: A discontinuous eigenfunction problem. Preliminary report.

Given the systems of differential equations $dy_i = F_{i1}y_1 + \ldots + F_{in}y_n$ $(i = 1,2,\ldots,n)$ where the F_{ij} are continuous functions of x and t for $a \leqq x \leqq b$, $t_1 \leqq t \leqq t_2$. The solution sets are sought for which $y_1,\ldots,y_m$ $(0 < m < n)$ are continuous but y_i $(i > m)$ have the discontinuities $y_i(x_j + 0) - y_i(x_j - 0)$

$= c_{ij1}y_1(x_j) + \ldots + c_{ijm}y_m(x_j)$ where $a < x_1 < x_2 < \ldots < x_s < b$. The solution sets are also required to satisfy the boundary conditions $\sum_{k=1}^{n}\{p_{ik}y_k(a) + q_{ik}y_k(b)\} = 0$, $i = 1,2,\ldots,n$. The corresponding eigenvalues $t_1, t_2, \ldots$ are the roots of a certain determinantal equation; their variation with the c_{ijk} is being investigated. (Received March 5, 1958.)

544-62. J. L. Walsh and H. G. Russell: Integrated continuity conditions and degree of approximation by polynomials or by bounded analytic functions.

If C is an analytic Jordan curve in the z-plane, a function f(z) can be approximated in the pth power ($p \geqq 1$) mean on C of order $O(n^{-k-\alpha})$ by polynomials in z of degree n when and only when f(z) is of class H_p (i.e. the transform of f(z) under conformal map of the interior of C onto $|w| < 1$ is of class H_p) and $f^{(k)}(z)$ satisfies on C a pth power integrated Lipschitz condition of order α ($0 < \alpha < 1$) or Zygmund condition ($\alpha = 1$). If D is a bounded region containing C and its interior, an equivalent necessary and sufficient condition is that f(z) can be approximated in the pth power mean on C of order $O(n^{-k-\alpha})$ by functions $f_n(z)$ analytic and with $|f_n(z)| \leqq R^n$ in D, where R is constant. (Received March 12, 1958.)

544-63. Louis Auslander: A class of discrete uniform subgroups of solvable Lie groups.

Let Z^s denote the additive group of integers taken s times and let Γ satisfy the exact sequence $1 \to Z^s \to \Gamma \to Z^t \to 1$. Since Z^s is a normal subgroup of Γ, Z^t acts as a group of automorphisms of Z^s and this action can be extended by linearity to give the action of Z^t on $Z^s \otimes R$, where R is the real field. A sufficient condition for Γ to be a discrete uniform subgroup of a connected, simply connected, solvable Lie group is that the action of Z^t on $Z^s \otimes R$ can be extended to the action of $Z^t \otimes R$ operating linearly on $Z^s \otimes R$. (Received March 3, 1958.)

544-64. Richard Stearns: Some results on preference patterns. Preliminary report.

(1) A point p of a connected preference pattern S (also called a connected complete digraph) is called a cut point if S-p is disconnected. An order relation is established among the cut points of an arbitrary connected pattern and all the patterns with zero or one noncut points are exhibited. (2) Let p_i be the

number of points b of S such that a_iPb. S is connected if and only if for all S' of S, $\sum a_i > C_{k,2}$ where the sum is over the k elements of S'. The number of three cycles in a pattern of order n is $(1/12)n(n-1)(2n-1) - 2^{-1}\sum_1^n p_i^2$. (3) There exists a one to one mapping $a_i \rightarrow t(a_i)$ of the points of any pattern S onto the transitive pattern of the same order such that the number of pairs (a_i,a_j) where a_iPa_j and $t(a_j)Pt(a_i)$ is no more than $(n-1)^2$ if S has 2n points or (n^2-n+1) if S has 2n + 1 points. (4) A pattern S is said to be generated by a set of transitive patterns T_k on the points a_i $(i = 1, \ldots, n)$ if a_iPa_j in S if and only if a_iPa_j in a majority of the T_k. The maximum number of T_k required to generate an arbitrary pattern is less than 4n/3. (Received March 3, 1958.)

544-65. B. L. Foster: Differentiation on manifolds without a connection.

On a differentiable manifold, M, of class C^3, let X^a, X^α, X^A be three different sets of coordinate variables; let $D^a_\alpha = \partial X^a/\partial X^\alpha$, $D^a_{\alpha\beta} = \partial X^a/\partial X^\alpha \partial X^\beta$, etc. The ordinary Jacobian, D^a_α, is denoted by J and a generalized transformation operator is defined by $K = (k_{ij})$, $1 \leqq i, j \leqq 2$, where $k_{11} = D^a_\alpha$, $k_{12} = D^a_{\alpha\beta}$, $k_{21} = 0$, and $k_{22} = D^a_\alpha D^b_\beta$. K is a transitive, linear, homogeneous operator, hence gives rise to a generalized notion of tensor, which permits the derivative of a tensor to be a tensor. For example, if T_a is an ordinary vector then $(T_a, \partial T_a/\partial X^b)$ is a tensor of type K. More generally, the pair consisting of any ordinary tensor and its partial derivative is a tensor whose type is a "Leibnitz-like" expansion of J's and K's. The set of all affine connections on M is a convex subset of the vector space of all tensors of type $K \otimes J^{-1}$. Also, direct sum decompositions are given, relating ordinary tensors and tensors of generalized types. (Received March 4, 1958.)

544-66. R. J. Mihalek: Modularity relations in lattices. Preliminary report.

For a lattice L with operations +, ·, and ordering $\leqq$, the modular relation is relativized as follows: for $S \subset L$, $(b,c)M_S$ means $a \in S$, $a \leqq c$ implies $(a + b)c = a + bc$. For a join-closed set S, a modularity and independence theory is obtained comparable to that of L. R. Wilcox (Modularity in the theory of lattices, Ann. of Math., April, 1939). These ideas are applied to the concept of weak-modularity in the study of quasi-dual-ideals. Some instances are studied in which the modularity of L is determined or partially determined by M_S for appropriate sets S. Finally, the set $\mathfrak{M}$ of modularity relations relative to

all the subsets of L forms a complete lattice when ordered set-theoretically. Some properties of $\mathcal{M}$ are studied including a Galois connection between the Boolean algebra of the set of all subsets of L and $\mathcal{M}$. (Received March 6, 1958.)

544-67. R. H. Rosen: Fixed points for multi-valued functions on snake-like continua.

A snake-like continuum is a compact metric chainable continuum. A multi-valued function $f:X \to Y$ is upper-semi-continuous (u.s.c.) if for each sequence (x_n) converging to a point x in X, $\lim_n \sup f(x_n) \subseteq f(x)$; lower-semi-continuous (l.s.c.) if $f(x) \subseteq \lim_n \inf f(x_n)$; and continuous if it is both u.s.c. and l.s.c. If f and g are multi-valued functions from X into Y and there is a point x in X so that $f(x) \cap g(x) \neq 0$, x is an incidence point for f and g; if X = Y and g is the identity map, x is a fixed point for f. Let all spaces be snake-like continua and all functions be multi-valued. If $f:S \to S$, it is proved that each of the following conditions is sufficient for f to have a fixed point: (1) f is u.s.c. and the graph of f is connected, (2) f is u.s.c. and f(x) is connected for each x in S, and (3) f is continuous. These results are generalized in terms of incidence points. One consequence of these generalizations is, (4) if f and g are maps from S into T and f is onto, then f and g have an incidence point. (Received April 2, 1958.)

544-68. P. C. Rosenbloom and D. V. Widder: Representation of solutions of the heat equation.

Polynomial solutions of the heat equation are studied, and necessary and sufficient conditions for the representation of a solution as a series in these special solutions, or their Appell transforms, are obtained. These results are connected with the conditions for the representability of a solution as a Poisson-Stieltjes integral and with the validity of Haygens' principle for the heat equation. Blackman's phenomenon is explained in the light of the present theory. (Received March 7, 1958.)

544-69. Morris Schreiber: Representation of operators.

Continuing the work of [Bull. Amer. Math. Soc. Abstract 63-2-257; see also Trans. Amer. Math. Soc. January 1958] we call a contraction A on Hilbert space absolutely continuous (AC) if its strong operator measure has an operator valued "derivative" K (with respect to Lebesgue measure on the unit circle C) such that $0 \leqq K(z) \leqq cI, |z| = 1$, c = constant depending on A, I = identity oper-

ator. This means that $A^n = \int_C e^{in\theta} K(e^{i\theta}) d\theta/2\pi$. Such operators have the following preservation of spectrum properties: (i) if α is an eigenvalue of A then given $f \in H^1$ = 1st Hardy class there exists a unit vector x such that $(f(A)x,x) = f(\alpha)$; (ii) if α is an approximate eigenvalue with $|\alpha| < 1$, then given $f \in H^1$, for any $\delta > 0$ there exists a unit vector x such that $(f(A)x,x) = f(\alpha) \pm \delta$. From these remarks there follow various corollaries. In particular, if the approximate point spectrum has a limit point inside C then the map $f \to f(A)$ is one-one from H^1 to the uniformly closed ring R generated by A and I, and if there is an eigenvalue on C then the map is a metric equivalence of H^2 and R. There is an application to the invertibility of Toeplitz matrices $T = (c_{i-j})$, or what is the same, the solution of the convolution equations $y_i = \sum_j c_{i-j} x_j$. An isolated comment: if A is (AC) and B = I + A, then B has nth roots and $\lim_n B^{1/n} = I$. (Received March 6, 1958.)

544-70. Hidehiko Yamabe: A deformation theorem on compact manifolds.

Let S be a C^6 compact d-dimensional manifold with Riemannian structures represented by $G_{ij}(x,t)$ with a parameter t subject to $\partial G_{ij}/\partial t = -2(U - \int U dV_t) G_{ij}$ where $U(x,t)$ denotes the scalar curvature corresponding to G_{ij}, dV_t the corresponding volume element. Then U must satisfy $\partial U/\partial t = 2(d-1)\Delta_t U + 2(U - \int U dV_t)U$ where Δ_t denotes the corresponding Laplace-Beltrami operator. From this, $\partial \int |U|^{d/2} dV_t/\partial t \leqq 0$, $\partial \int U dV_t/\partial t = -2(d-2)/(U - \int U dV_t)^2 dV_t$, $G_{ij}(t) = -2\int_0^t (U - \int U dV_t) dt G_{ij}(0)$. Using these, the existence of solutions of the equation is proved for any continuous initial value, which approaches to a constant. Then the limit Riemannian structure has the constant scalar curvature. The proof can be shortened if L_p type Dirichlet principle is available. (Received March 6, 1958.)

544-71. Frank Carroll: On a class of noncontinuable analytic functions.

(I) Let α, θ, δ be real, $\delta > 0$, $0 \leqq \theta < \pi$, $F(z) = \sum_{n>\alpha} a(n) z^n$ converge for $0 < |z| < 1$. A NASC that F(z) define an analytic function for $\theta < \arg z < 2\pi - \theta$ with $F(z) = O(z^{\alpha+\epsilon})$ uniformly in the sector $\theta + \epsilon \leqq \arg z \leqq 2\pi - \theta - \epsilon$ for each $\epsilon > 0$ is: $\exists$ a function $g(w) = g(u + iv)$ with $g(n) = a(n)$ $(n = [\alpha] + 1, [\alpha] + 2, \ldots)$ $\ni (*) \forall \epsilon > 0$, $g(w)$ is analytic for $u \geqq \alpha + \epsilon$, where $g(u + iv) = O(\exp(\delta|u| + (\theta + \epsilon)|v|))$. Let g(w) satisfy (*) for $\theta = 0$. (II) If $\sum_{n=0}^{\infty} b(n)g(n)z^n$, $\sum_{n=0}^{\infty} g(n)z^n$, and $\sum_{n=0}^{\infty} b(n)z^n$ have radius of convergence 1, and the first has $|z| = 1$ as N.B. (natural boundary), then so does the third. (III) For $x > 0$, let f(x) be real,

$|x^{2r}f''(x)| < K$, $(r > 0)$, $f'(n) \pmod{2\pi}$ $(n = 1,2,\ldots)$ dense in $[0,2\pi]$. Let $a(n) > 0$, $a(n) \to 0, |a(n)n^{r}| > Y > 0$. Then $\sum_{1}^{\infty} n^{\rho} a(n)e^{if(n)}z^{n}$ has $|z| = 1$ as N.B. Let $h(n) = \prod_{1}^{m} \Gamma(c_k n + d_k)^{\delta_k}$ where $\delta_k = \pm 1, c_k, d_k$ are complex constants, $\mathrm{Re}(c_k) \geqq 0$. Consider the generalized hypergeometric series $H(z) = \sum_{n'} h(n)z^{n}$ $(n' \geqq 0, h(n') \neq \infty)$. Let $K = -\sum \delta_k c_k$. H(z) converges only at $z = 0$ if $\mathrm{Re}(K) < 0$, is entire if $\mathrm{Re}(K) > 0$, and has a radius of convergence $R(0 < R < \infty)$ if $\mathrm{Re}(K) = 0$. Using (II), (III) applied for $f(x) = Ax \log x$, and Stirling's formula, one obtains (IV). If $K = 0$ then H(z) represents a function single-valued and analytic in the complement of a half-ray; if $\mathrm{Re}(K) = 0, K \neq 0$, then H(z) has $|z| = R$ as its N.B. (Received March 3, 1958.)

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545-29. C. B. Bell: On the structure of stochastic independence.

Let $\{(X,M_t,\mu_t)\}$ $(t \in \hat{T})$ be an arbitrary at-least-countable family of probability spaces; and let each $\bigcap A_{t_j}$ be such that $A_{t_j} \in M_{t_j}$ for all j and $t_i \neq t_j$ for $i \neq j$. What are the relations between the following four independence conditions? (C_0) $\bigcap A_{t_j} \neq \emptyset$ whenever $A_{t_j} \neq \emptyset$ for all j. (C_1) There exists a measure μ on $S(\bigcup M_t)$ such that $\mu(\bigcap A_{t_j}) = \prod \mu_{t_j}(A_{t_j})$. (C_2) $\bigcap A_{t_j} \neq \emptyset$ whenever $\mu_{t_j}(A_{t_j}) \neq 0$ for all j. (C_3) $\bigcap A_{t_j} \neq \emptyset$ whenever $\prod u_{t_j}(A_{t_j}) \neq 0$. One obtains the solution by studying the intersection of the diagonal D of the product space $X_t(X,M_t,\mu_t)$ with the product σ-algebra; the sets of positive measure; the rectangles of positive measure; and the rectangles with all sides of positive measure. It is shown here that $(C_2) \rightarrow (C_1)$. From several counterexamples here, and the work of Banach and Sikorski, one concludes that no two conditions are equivalent and that $(C_0) \rightarrow (C_2) \rightarrow (C_1) \rightarrow (C_3)$. (Received April 7, 1958.)

545-30. Alfred Horn and Robert Steinberg: Eigenvalues of the unitary part of a matrix.

The pair of n-tuples (λ_j), (α_j), with $\lambda_j \neq 0, |\alpha_j| = 1$ for all j, is called realizable if there exists an $n \times n$ matrix A with polar decomposition $A = PU$ (with P positive and U unitary) such that the eigenvalues of A and U are the terms of (λ_j) and (α_j) respectively. If exp ia, exp ib are terms of a sequence of complex numbers of modulus 1, if $a < b < a + \pi$, and if $0 \leq 2d \leq b - a$, then the operation of replacing the pair exp ia, exp ib by the pair exp i(a + d), exp i(b - d) is called a pinch. Theorem. The pair of n-tuples (λ_j), (α_j), with $\lambda_j \neq 0$, $|\alpha_j| = 1$ for all j, is realizable if and only if there is a finite sequence of pinches leading from (α_j) to some arrangement of $(\lambda_j/|\lambda_j|)$. An equivalent condition can be stated in terms of inequalities relating suitable determinations of $(\arg \lambda_j)$, $(\arg \alpha_j)$. (Received February 28, 1958.)

545-31. R. R. Phelps: Some subreflexive Banach spaces.

Call a Banach space E subreflexive if the collection P of all those linear functionals in E^* attaining their supremum on the unit cell of E forms a norm-dense subset of E^*. If E is reflexive then $P = E^*$ so that E is subreflexive.

There is no known example of a nonsubreflexive Banach space. (In fact, it is unknown whether P is always weakly dense in E*.) It is shown that every (AL)-space and every (AM)-space with unit is subreflexive. These results depend heavily on the concrete representations of such spaces given by Kakutani [Ann. of Math. vol. 42 (1941) pp. 523-537 and 994-1024] and, to a lesser extent, on the fact that an ℓ_1-product of subreflexive spaces is subreflexive [Phelps, Subreflexive normed linear spaces, to appear in Arch. Math.]. Using the above results together with other theorems by Kakutani it can be concluded that $\ell_1(S)$, $m(S)$, and all succeeding conjugate spaces are subreflexive. The same is true for $C(X)$ (the space of all bounded continuous real functions on the Hausdorff space X) as well as for $L_1(S)$, S a not-necessarily-σ-finite measure space. A slightly different proof shows that $C_0(X)$ (the space of all continuous real functions which vanish at infinity on the locally compact Hausdorff space X) and all its conjugates are subreflexive. (Received March 17, 1958.)

545-32. A. W. Marshall: A multivariate generalization of Chebyshev's inequality. Preliminary report.

The following generalization of Chebyshev's inequality has been derived: let $X_1, X_2, \ldots, X_n$ be random variables such that $E(X_i) = 0$, $E(X_i^2) = \sigma_i^2 < \infty$, $i = 1,2,\ldots,n$, and $E(X_i X_{i+1}) = \phi_i$, $i = 1,2,\ldots,n-1$. If $\epsilon_i > 0$, $i = 1,2,\ldots,n$, then $P(|X_i| < \epsilon_i,\ i = 1,2,\ldots,n) \geq 1 - 2^{-1}[\sigma_1^2/\epsilon_1^2 + \sigma_n^2/\epsilon_n^2 + \sum_{i=1}^{n-1}((\sigma_i^2/\epsilon_1^2 + \sigma_{i+1}^2/\epsilon_{i+1}^2)^2 - 4\phi_i^2/\epsilon_i^2\epsilon_{i+1}^2)^{1/2}$. (Received April 14, 1958.)

545-33. J. R. M. Radok: Method of functional extrapolation for the numerical integration of ordinary differential equations.

The method of functional extrapolation leads to approximate solutions of systems of first order ordinary differential equations in terms of linear combinations of linearly independent functions which involve arbitrary parameters and which have sectionally continuous analytic derivatives. The coefficients and parameters in these approximations are determined in such a way that their values agree at the points of contiguity of the segments of the approximate solutions, while a certain number of their derivatives at the beginning of each segment are equal to the derivatives, obtained by substituting the approximate values of the dependent variables in the original differential equations and their derivatives. (Received March 6, 1958.)

545-34. E. T. Poulsen: On the Dirichlet problem for a nonuniformly elliptic differential operator.

Let $\mathcal{A} = -\partial/\partial x_i(a_{ij}(x)\partial/\partial x_j) + a_0$ be a formally self-adjoint elliptic operator on an arbitrary region $\Omega \subset E^m$ such that $a_{ij}, \partial a_{ij}/\partial x_i$ and a_0 are bounded in Ω. Let $D(\Omega)$ denote the restriction of $\mathcal{A}$ to the space $C_0^\infty(\Omega)$ of infinitely differentiable functions with compact support in Ω. The Friedrichs extension $\tilde{D}(\Omega)$ of $D(\Omega)$ is denoted the Dirichlet operator associated with $\mathcal{A}$ on Ω. The following theorem is proved: If $x \in \bar{\Omega}$ (including ∞ if Ω is unbounded) and U is a neighborhood of x, then there exists a neighborhood V of x, such that on $V \cap \Omega$ every $u \in D_{\tilde{D}(\Omega)}$ coincides with some function in $D_{\tilde{D}(\Omega \cap U)}$. If $H_D(\Omega)$ denotes the completion of C_0^∞ in the norm $\int_\Omega \{a_{ij}(\partial u/\partial x_j)(\partial \bar{u}/\partial x_i) + |u|^2\}dx$, it is first observed that $u \in D_{\mathcal{D}}$ iff $u \in H_D$ and the distribution $\mathcal{D}u \in \mathcal{L}^2$. It is then proved that if ω is infinitely differentiable with bounded derivatives and $u \in H_D$, then $\omega u \in H_D$, and further that if $u \in D_{\tilde{D}}$, then $\mathcal{D}(\omega u) \in \mathcal{L}^2$ so that $\omega u \in D_{\tilde{D}}$, and the result follows. The theorem remains true if the points of $\bar{\Omega}$ in the neighborhood of which a_{ij} and $\partial a_{ij}/\partial x_i$ are not bounded and a_0 is not bounded above form a totally disconnected set. (Received March 6, 1958.)

545-35. P. F. Conrad: Halfgroupoid rings.

Let Γ be the set of all mappings of a halfgroupoid S into a ring R, and let $G = [\pi \in \Gamma : s\pi = 0$ for all but a finite number of $s \in S]$. If $R^3 \neq 0$, then G is a ring if and only if for $a,b,c \in S$, (i) ab, (ab)c $\in$ S if and only if bc, a(bc) $\in$ S, and (ii) if ab, (ab)c $\in$ S, then (ab)c = a(bc). If S also admits a partial order $\leqq$ that satisfies, (1) if $w \leqq x \leqq z$ and $w \leqq y \leqq z$, then $x \leqq y$ or $y \leqq x$, and (2) if wy, xz $\in$ S, $w \leqq x$ and $y \leqq z$, then $wy \leqq xz$ and the equality holds if and only if $w = x$ and $y = z$, then "all but a finite number" can be replaced by "all but a finite set of well ordered sets". Seven different extensions of G in Γ are studied. In this framework we recover function rings, rings of (finite or infinite) matrices, polynomial rings, rings of formal power series, and generalizations of these concepts. Matrix rings are obtained if and only if S satisfies, (a) for each pair $a,b \in S$ there exist $x,y \in S$ such that $xay = b$, and (b) if xay, xby $\in$ S, then $a = b$. By linearly ordering this last type of halfgroupoid matrix rings are obtained that are not row or column finite. (Received March 6, 1958.)

545-36. Louis Weinberg and Paul Slepian: Positive real matrices.

It is well known that the positive real (p.r.) concept is one of the most important in network theory. Its importance derives from the following two facts: (1) A necessary and sufficient condition for a real rational function to be realizable as the driving-point impedance of a one-port network is that is be a p.r. function. (2) A necessary and sufficient condition for a symmetric nth-order matrix of real rational functions to be realizable as the open-circuit impedance matrix of an n-port network is that it be a p.r. matrix. Since the definitions of a p.r. rational function and a p.r. matrix of rational functions are not suitable for testing purposes, equivalent sets of necessary and sufficient conditions that are more suitable as tests have been presented in the literature. All of these tests require, in general, the solving of real polynomials for real roots. In this paper new sets of necessary and sufficient conditions for a rational function and a matrix of rational functions to be p.r. are formulated. These conditions eliminate root solving and give insights that may be useful in research on unsolved synthesis problems. (Received March 6, 1958.)

545-37. Jun-ichi Hano: On complex homogeneous spaces of compact Lie groups.

Let G be a connected compact Lie group and let B be a connected closed subgroup of G. We call B a C-subgroup when there is a toral subgroup T of G such that the semi-simple part of B coincides with that of the centralizer of T, according to C. Wang. We establish the following: Theorem 1. When dim G/B is even, G/B admits an invariant complex structure if and only if B is a C-subgroup. The question of how an invariant complex structure is defined is clarified from the following: Theorem 2. If G/B admits an invariant complex structure, the real dimension of the vector space of all G-invariant conformal vector fields is equal to rank G-rank B. These results give not only a generalization of C. Wang's theory on a complex compact homogeneous space with finite fundamental group, but also a different approach to his theory. (Received March 6, 1958.)

545-38. G. B. Preston: On a semigroup of order isomorphisms determined by an ordered semigroup.

Let T be a totally ordered multiplicative semigroup such that: (1) the multiplication in T is continuous in the order topology (2) T is connected in the

order topology; (3) T is bounded with minimal element 0 and maximal element 1; (4) 0 is a multiplicative zero and 1 is a multiplicative identity for the semigroup T. Then T is said to be a standard thread (Cf. A. H. Clifford, Bull. Amer. Math. Soc. vol. 63 (1957) p. 277 and Trans. Amer. Math. Soc., to appear). We will say that two continuous semigroups are topologically isomorphic if there exists a homeomorphism between them which is also an isomorphism. A standard thread T determines a set $\mathscr{S}(T) = \{\lambda : \lambda$ is a topological isomorphism of a convex subsemigroup of T onto a convex subsemigroup of $T\}$. The elements of $\mathscr{S}(T)$ can be regarded as subsets of $T \times T$ and we adjoin the empty subset to $\mathscr{S}(T)$ to form the set S(T). Then under the usual operation of composition of subsets of a set $T \times T$ the set S(T) becomes an inverse semigroup (Cf. J. London Math. Soc. vol. 29 (1954) pp. 411-419). We prove the following theorem: if T_1 and T_2 are two standard threads then T_1 is topologically isomorphic to T_2 if and only if $S(T_1)$ is isomorphic to $S(T_2)$. (Received March 7, 1958.)

545-39. D. H. Lehmer: On the exact number of primes less than a given limit.

The problem of enumerating the primes $\leq x$ without listing them all and counting the list was first discussed by Legendre in 1798 who used a formula involving all the primes not exceeding the square-root of x. In 1870 E. Meissel gave a modification of Legendre's formula in which only the primes less than the cube-root are needed. He used this formula to determine the exact number of primes less than various limits up to 10^9. In the present paper a simpler method of looking at the problem is presented and a general modification is obtained involving the primes not exceeding the r-th root of x. For $r = 4$ the method reaches its maximum of practicality. Examples of the formula are exhibited. The oft-quoted result of Meissel for $x = 10^9$ is shown to be too small by 56. The number of primes less than 10^{10} is 455052 512. Comparisons are made with $\ell_i(x)$ and the formula of Riemann. (Received March 7, 1958.)

545-40. Herman Rubin: Convergence of distributions on function spaces.

Let T be a bounded closed interval and let F be the class of all functions on T with discontinuities only of the first kind and continuous at the endpoints. Let U_{gh} be the class of all elements f of F such that for some $\epsilon > 0$, $g + \epsilon < f < h - \epsilon$. Then the topology $\mathcal{J}$ with base $\{U_{gh} : g,h$ continuous$\}$ is separable metric, and hence if Q_n, P are probability distributions on F such

that the random function has a probabilistic modulus of continuity uniform in n, and the m-dimensional marginal distributions under Q_n converge to those under P, then Q_n converges to P in the weak topology generated by $\mathcal{J}$. An immediate special case is the extension of Kimme's results(Trans. Amer. Math. Soc. vol. 84 (1957) p. 208) to functionals which are $\mathcal{J}$-continuous with P-probability 1. (Received March 10, 1958.)

545-41. W. J. Firey and D. Bushaw: Invariant arcs of certain mappings having two fixed points.

Let M be a mapping with Lipschitzian derivatives of a closed disc D in the Euclidean plane. If $MD \subseteq D$ and there are two fixed points on the boundary B of D, then there is a simple continuous arc A joining the fixed points such that $MA = A$. The proof involves reduction to the case $MD = D$. Then either one of the arcs of B joining the fixed points in A or M reverses orientations. In this case there is a field of invariant directions in D and a tangent curve to this field through one of the fixed points is A. (Received March 10, 1958.)

545-42. D. W. Bushaw: The equation of damped vibrations.

General results are obtained about qualitative properties of solutions of the differential equation $\ddot{x} + g(x,\dot{x}) + h(x) = 0$, where g(x,y) and h(x) are continuous and have the signs of y and x respectively. The principal results are: (1) Every solution can be continued over an interval of the form $[t_0,\infty)$. (2) The equilibrium solution $x \equiv 0$ is globally asymptotically stable, provided that the integral of h(x) is divergent in each direction. Simple direct proofs are given for the criterion distinguishing the oscillatory from the nonoscillatory case on the assumption that g(x,y) + h(x) is differentiable. (Received March 10, 1958.)

545-43. Philip Wolfe: Simplex method for quadratic programming. II.

Given matrices: A, m by n; b, m by 1, $b \geqq 0$; C, m by n, positive definite, symmetric. To solve for all $\lambda \geqq 0$: Minimize $x'Cx/2$ subject to $x \geqq 0$, $Ax = b$. Let ' denote transposition, $x = (x_1,\ldots,x_n)'$, $y = (y_1,\ldots,y_m)'$, $u = (u_1,\ldots,u_m)$, $z = (z_1,\ldots,z_n)'$, $s = (s_1,\ldots,s_m)'$, $t = (t_1,\ldots,t_n)'$. Solve the following linear program under the restriction: do not allow x_j in the basis if v_j is in ($j = 1,\ldots,n$), and vice versa. (1) Minimize $\sum s_i + \sum t_j$ to zero subject to (i) $x,y,v,z,s,t \geqq 0$, (ii) $Ax + y + s = b$, (iii) $Cx - uA' - v + z + t = 0$; (2) Minimize $\sum y_i$ to zero subject to (i) - (iii) and (iv) $s,t = 0$; (3) Minimize $\sum z_j$ to zero subject to (i) - (iv)

and (v) $z = 0$; (4) Maximize λ to ∞ subject to (i,ii,iv,v) and (vi) $Cx - uA' - v + \lambda p = 0$. The x associated with any value of λ obtained in the course of solving (4) solves the stated quadratic problem. (Received March 20, 1958.)

545-44. H. S. Bear: A strong maximum modulus property for maximal function algebras.

Let C(X) be the algebra of all continuous complex-valued functions on the compact Hausdorff space X, and A be a maximal closed subalgebra of C(X), containing the constants and separating points of X. A may be considered as an algebra of functions on its maximal ideal space Σ and X as a subset of Σ. Theorem: If $f \in A$ and $|f(x_0)| = \|f\|$ for some $x_0 \in \Sigma \sim X$, then f is constant. (Received March 12, 1958.)

545-45. Haskell Cohen and I. S. Krule: Continuous homomorphic images of real clans with zero.

Let S be a clan with zero, z, whose underlying topological space is the closed interval of real numbers $[\delta,u]$, where u is the unit of S. Such a clan is called a real clan with zero. That u must be an endpoint of S is well known. Let h be a continuous homomorphism of S onto the clan T. Using the characterization of the multiplicative structure of S [P. S. Mostert and A. L. Shields, On the structure of semigroups on a compact manifold with boundary, Ann. of Math. vol. 65 (1957) pp. 117-143] and [Haskell Cohen and L. I. Wade, Clans with zero on an interval, Trans. Amer. Math. Soc., to appear] and the cutpoint properties of S, the following result is established: Either (1) T is a real clan with zero and its endpoints are h(u) and h(z) or h(u) and $h(\delta)$, or (2) T is a triad and its endpoints are h(u), $h(\delta)$, and h(z). An example of (2) is furnished, and it is observed that the cutpoint properties of T are independent of the metrisability of S. (Received March 28, 1958.)

545-46. E. A. Michael: Convex structures and continuous selections.

A convex structure on a metric space permits one to take "convex combinations" of certain admissible n-tuples in a suitably consistent and continuous fashion. A subset S of such a space is called convex if every n-tuple in S is admissible, and if every convex combination of such an n-tuple is in S. An interesting example is provided by a metrizable group G (left invariant metric) and a closed subgroup H which is a Banach space; an n-tuple is admissible if

it lies in a right coset of H, while convex combinations are defined as usual on H, and then by right translation on the cosets. In this example, all right cosets of H are convex. The following result generalizes the principal result of the author's [Ann. of Math. vol. 63 (1956) pp. 361-382]. THEOREM: Let X be paracompact, Y a metric space with convex structure, C(Y) the family of nonempty, complete convex subsets of Y. Then every lower semi-continuous $\phi: X \to C(Y)$ admits a selection. This theorem also has a "local version" which is the principal tool in the proof of the cross-section theorem announced by the author in [Abstract 539-28, Notices Amer. Math. Soc. vol. 5 (1958) p. 66]. (Received March 6, 1958.)

545-47. Albert Cahn and E. G. Straus: On piece-wise congruence. Preliminary report.

The following relations are considered between sets A, B in a space S on which a group of motions G is acting: (1) $A \cong B$ if $\phi(A) = B$ for some $\phi \in G$. (2) $A \cong_n B$ if $A = A_1 \cup A_2 \cup \ldots \cup A_n$, $B = B_1 \cup B_2 \cup \ldots \cup B_n$ with $A_i \cap A_j = B_i \cap B_j = 0$ for $i \neq j$ and $A_i \cong B_i$ $(i = 1,\ldots,n)$. (3) $A \cong_{(f)} B$ if $A \cong_n B$ for some finite n. (4) $A \cong_{n,m} B$ if $A \cong_n A_0 \cong_n A_1 \cong_n \cdots \cong_n A_m = B$. (5) $A \cong_{n,(f)} B$ if $A \cong_{n,m} B$ for some finite m. All these relations are reflexive and symmetric but only $\cong$, $\cong_{(f)}$, $\cong_{n,(f)}$ are always transitive. Cardinality is invariant under all these relations, which were originally considered in the study of area in the Euclidean plane. The following theorems hold. (6) $A \cong_{n,m} B \Rightarrow A \cong_{p,q} B$ if $n \leq p$, $m \leq q$. (7) $A \cong_n B \cong_p C \Rightarrow A \cong_{np} C$. (8) $A \cong_n B$ and $A \cap B = 0 \Rightarrow A \cong_{2,n} B$. Negations of implications can hold only if G is specified, since the congruences (1) - (5) are equivalent for $G = \{I\}$ or G the full symmetric group on S. For G = S = additive group of integers, one has the following. (9) $A \cong_n B \not\Rightarrow A \cong_{n-1} B$ for $n > 1$. (10) $A \cong_n B \cong_p C \not\Rightarrow A \cong_{np-1} B$ for $np > 1$. (11) $A \cong_{n,m} B \not\Rightarrow A \cong_{n,m-1} B$ for $n > 1$. Sierpiński (On the congruence of sets and their equivalence by finite decomposition, Lucknow) has asked the following questions for Euclidean space: Do the implications $A \cong_3 B \Rightarrow A \cong_{2,2} B$ or $A \cong_{(f)} B \Rightarrow A \cong_{2,(f)} B$ hold? We intend to discuss these questions. (Received March 7, 1958.)

545-48. Klaus Krickeberg: Semimartingales going down.

Let μ be a strictly positive and finite measure on a Boolean sigma-algebra $\mathfrak{L}$ with unit E, θ a set directed by a transitive relation $\ll$, and $(\mathfrak{L}_\tau ; \tau \in \theta)$ a

decreasing Moore-Smith sequence of sigma-subalgebras of $\mathcal{L}$. A sequence $(f_\tau;\ \tau \in \theta)$, where f_τ is a $\mathcal{L}_\tau$-measurable and integrable real function on E, is called a semimartingale if $\rho \ll \tau$ and $A \in \mathcal{L}_\tau$ implies $\int_A f_\rho d\mu \geqq \int_A f_\tau d\mu$, and a martingale if equality holds here. Every semimartingale converges stochastically. A semimartingale (f_τ) in $\mathfrak{L}^p = \mathfrak{L}^p(\mu)$, $1 \leqq p < +\infty$, converges strongly in $\mathfrak{L}^p$ if there is a martingale (g_τ) in $\mathfrak{L}^p$ such that $f_\tau \geqq g_\tau$ for every τ, and $\lim_\tau \int_E f_\tau d\mu > -\infty$ alone is sufficient for strong convergence in $\mathfrak{L}^1$. A scale of Vitali covering conditions V' and V_q about $(\mathcal{L}_\tau)$ can be defined for $1 \leqq q \leqq +\infty$ such that V' is satisfied if $\ll$ is simple, V' implies every V_q, and V_q implies V_r if $r < q$. The essential convergence of every martingale in $\mathfrak{L}^p$ follows from V_q if $p^{-1} + q^{-1} = 1$, and V' implies the essential convergence of all semimartingales. $V_{+\infty}$ and V' are analogues to the conditions V_0 and V' about an increasing sequence $(\mathcal{L}_\tau)$ defined in Trans. Amer. Math. Soc. vol. 83 (1956) p. 325 and Math. Z. vol. 66 (1957) p. 486. (Received January 21, 1958.)

545-49. Donald Greenspan: On the numerical solution of three dimensional Dirichlet problems.

The three dimensional Laplace equation is replaced by the finite difference analogue: $-2u(x,y,z) + h_2^2h_3^2A[u(x + h_1,y,z) + u(x - h_1,y,z)] + h_1^2h_3^2A[u(x,y + h_2,z) + u(x,y - h_2,z)] + h_1^2h_2^2A[u(x,y,z + h_3) + u(x,y,z - h_3)] = 0$, where $A = (h_1^2h_2^2 + h_2^2h_3^2 + h_1^2h_3^2)^{-1}$. The finite difference analogue is used to produce a system of linear, algebraic equations whose solution constitutes the essential part of the numerical solution. It is shown that the numerical solution is unique and that under general conditions it converges to the analytic solution. (Received February 20, 1958.)

545-50. C. M. Fulton: Linear completeness and hyperbolic trigonometry.

Any model for hyperbolic geometry allows the derivation of a relation between a segment and its angle of parallelism. In this paper Hilbert's postulate of linear completeness is used to show the uniqueness of this relation. A generalized relation is then found for two interior angles of parallelism, namely $e^{-c} = \tan \alpha/2 \tan \beta/2$. Through a vertex of a right triangle the two parallels are drawn to the opposite side. Applying the above relation to the hypotenuse, two basic trigonometric formulas can now be derived. (Received February 18, 1958.)

545-51. C. W. Clark: Asymptotic distribution of the eigenvalues for the lower part of the n-dimensional Schrödinger operator spectrum.

Consider the eigenvalue problem $-\nabla^2 u(x) + V(x)u(x) = \lambda u(x)$, $x \in R_n$, $u(x) \in L_2(R_n)$. For $N(\lambda) = \sum_{\lambda_j \leqq \lambda} 1$, the number of eigenvalues $\leqq \lambda$, the formula $N(\lambda) \sim \{[2n^{1/2}]^n \Gamma(n/2 + 1)\}^{-1} \times \int_{V(x) \leqq \lambda} [\lambda - V(x)]^{n/2} d\mu_n(x)$ has been established by Titchmarsh, Ray, and others, under various conditions on $V(x)$, all of which assume that $V(x) \to +\infty$ as $|x| \to +\infty$. As reported by Brownell [Bull. Amer. Math. Soc. Abstract 62-4-566 and 63-4-559], the formula can remain valid for the negative eigenvalues $\{\lambda_j\}_{j=1}^{\infty}$ in the case that $V(x) \to h$ (h finite) as $|x| \to +\infty$. In particular, the formula can be established for radial functions $V(x) = q(|x|)$ provided that $q(|x|)$ satisfies the conditions quoted in Abstract 545-11 Notices Amer. Math. Soc. vol. 5 (1958) p. 226, as well as (iv) there exists η, $0 < \eta < 2$, and $\gamma, 2 - \eta \geqq \gamma > (2 - \eta)\{1 + \eta^2/[4(n + 3)\eta + 4]\}^{-1}$, such that $M_1|x|^{-(2-\eta)} \leqq |q(|x|)| \leqq M_2|x|^{-\gamma}$ over $|x| \geqq x_0$ for some x_0. For non-radial $V(x)$, the asymptotic formula will follow if $V_1(x) \leqq V(x) \leqq V_2(x)$ over $|x| > x_0$, where $V_1(x)$ and $V_2(x)$ are radial satisfying the above conditions, and are such that the formulae for V_1 and V_2 are asymptotically equal (in addition, $V(x)$ must satisfy conditions over $|x| \leqq x_0$ as before). This latter would be assured, for example, if $|V_1(x) - V_2(x)| = O(|x|^{-2})$. (Received February 28, 1958.)

545-52. H. S. M. Coxeter: Twelve points in PG(5,3) with 95040 self-transformations.

The finite projective space PG(5,3) (of five dimensions, over the Galois field with 3 elements) contains $364 = (3^6 - 1)/2$ points, from which 12 can be selected so as to have the following properties. Every 5 of the 12 determine a hyperplane (4-space) which contains just one more point of the set. Thus there are 132 hyperplanes, each containing 6 of the 12 points. This is a geometrical realization of the Steiner system $\mathcal{S}(5,6,12)$, whose group is known to be the quintuply transitive Mathieu group M_{12}, the only simple group of order 95040. In any one of the hyperplanes, the 6 points can be divided into two triads in 10 ways. The two triads determine two planes, which have just one point of intersection. It is found that the 10 points so constructed all lie in a 3-space, and form a nonruled quadric in PG(3,3). (Received March 5, 1958.)

545-53. Richard Montague: Incomparable theories.

For terminology see Tarski, Mostowski, Robinson, Undecidable theories;

in particular, P is Peano's arithmetic, and Q is Robinson's finitely axiomatizable subtheory of P. A set K of sentences is called strongly independent if, for each $\phi \in K$, the theory generated by ϕ is not relatively interpretable in $K \sim \{\phi\}$. Theorem. If T is any finitely axiomatizable subtheory of P and n is any positive integer, then there is a set K of n sentences such that each member of K logically implies all theorems of T, K forms a subtheory of P, and K is strongly independent. The proof depends partly on finding n sentences $\phi_1,\dots,\phi_n$ such that, for $1 \leqq i \leqq n$, $\vdash_Q \phi_i \leftrightarrow \bigwedge x[\rho(x,\overline{\phi_i}, \phi_1 \wedge \dots \wedge \phi_{i-1} \wedge \phi_{i+1} \wedge \dots \wedge \phi_n) \rightarrow \bigvee t\,(t < x \wedge [\rho(t,\overline{\phi_1}, \phi_2 \wedge \dots \wedge \phi_n) \vee \dots \vee \rho(t,\overline{\phi_{i-1}}, \phi_1 \wedge \dots \wedge \phi_{i-2} \wedge \phi_i \wedge \dots \wedge \phi_n) \vee \rho(t,\overline{\phi_{i+1}}, \phi_1 \wedge \dots \wedge \phi_i \wedge \phi_{i+2} \wedge \dots \wedge \phi_n) \vee \dots \vee \rho(t,\overline{\phi_n}, \phi_1 \wedge \dots \wedge \phi_{n-1})])]$. Here $\overline{\phi}$ is the numeral corresponding to the Gödel number of a sentence ϕ, and $\rho(x,y,z)$ is a primitive recursive formula such that $\vdash_Q \bigvee x \rho(x,\overline{\phi},\overline{\psi})$ if and only if $T + \{\phi\}$ is relatively interpretable in $T + \{\psi\}$. Corollary. There are two finitely axiomatizable subtheories of P neither of which is relatively interpretable in the other. The corollary answers a question of Tarski. (Received March 3, 1958.)

545-54. Stefan Bergman: A class of pseudo-conformal and quasi-pseudo-conformal mappings.

Let a schlicht pseudo-conformal mapping $Z_k = Z_k(z_1,z_2)$, $z_k = x_k + iy_k$, $k = 1,2$, defined in $\tau = [0 \leqq x_k \leqq 1, 0 \leqq y_k \leqq 1]$ map τ onto a domain $\mathfrak{R}$ of the X_1,Y_1,X_2,Y_2 space. Then the inequality $\int_0^1\int_0^1 [L(y_1,y_2)]^2\, dY_1 dY_2 \leqq \mathrm{Vol}(\mathfrak{R})$ holds. Here $L(y_1^{(0)},y_2^{(0)})$ is the B-area of the image of the rectangle $[0 \leqq x_k \leqq 1, y_k = y_k^{(0)} = \text{const}, k = 1,2]$ in the X_1,Y_1,X_2,Y_2 space, see Mémor. Sci. Math. vol. 106, p. 9 and vol. 108. If for every y_k, $0 \leqq y_k \leqq 1$, we have $L(y_1,y_2) \geqq c$, then $c^2 \leqq \mathrm{Vol}(\mathfrak{R})$. For the area of the surface of an element formed by the vectors dS_k, $k = 1,2$, which form with each other the angle ψ the relation $dA = dS_1\, dS_2 \sin\psi$ holds, while for the B-area $dB = dS_1\, dS_2 \sin\chi$, where χ is the analytic angle between the above directions. Thus $dB = dA(1 - \sin^2\phi/\sin^2\psi)^{1/2} \cos\phi$ where ϕ is the angle of deviation. In addition to pseudo-conformal mappings one can consider mappings of extended class, i.e. mappings of the four-dimensional space by four functions $X_k(x_1,y_1,x_2,y_2)$, $Y_k(x_1,y_1,x_2,y_2)$, $k = 1,2$, satisfying the inequality $|\partial(X_1 + iY_1, X_2 + iY_2)/\partial(x_1,x_2)|^2 \leqq CT$, $T = [\partial(X_1,Y_1,X_2,Y_2)/\partial(x_1,y_1,x_2,y_2)]$, $C = C(x_1,y_1,x_2,y_2) > 0$ (quasi pseudo-conformal mappings). Then: $\int_0^1\int_0^1 [L(y_1,y_2)]^2\, dY_1\, dY_2 \leqq \int_0^1\int_0^1\int_0^1\int_0^1 CT\, dx_1 dy_1 dx_2 dy_2$. (Received April 28, 1958.)

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546-44. Frank Harary: On the number of dissimilar graphs between a given graph-subgraph pair.

The purpose of this paper is to integrate the theorems on enumerating the subgraphs and supergraphs of a given graph [Pacific J. Math. vol. 6 (1956) pp. 57-64 and vol. 7 (1957) pp. 903-911] by generalizing to a result which includes both of these as special cases. The enumeration method is again that of Pólya [Acta Math. vol. 68 (1937) pp. 145-254]. The combinatorial observation which accomplishes this generalization is that the appropriate configuration group for enumerating the dissimilar graphs between a graph G and a given subgraph H is the line group of G - H subject to the auxiliary condition that H is left invariant. (Received January 23, 1958.)

546-45. D. L. Kreider: On sets in both two-function-quantifier forms.

The recursion theorem in a particularly useful form given by Rogers and alteration-of-quantifiers techniques of Kleene and Addison are used to show that (i) the class C of notations for ordinal numbers (the broadest class of notations defined by Church and Kleene in their first paper on constructive ordinals, pp. 20-21 of Formal definitions, Fund. Math. vol. 28 (1937) pp. 11-21) is expressible in both 2-function-quantifier forms (of the Kleene analytic hierarchy), (ii) every set of non-negative integers obtainable by repeated application to the empty set of the hyperjump operation $A^h = \{a\}(\forall\, d)(\exists x)T_1^{A,d}(a,a,x)\}$ and of extensions of this operation through the class C is expressible in both 2-function-quantifier forms, and (iii) certain broad extensions of C as well as sets obtainable by application of the hyperjump operation through these extensions are still expressible in both 2-function-quantifier forms. (Received February 11, 1958.)

546-46. L. W. Small: Relation between recurrent events and an inductive law.

J. E. Freund, in the Amer. Math. Monthly, December, 1957, derives an expression for determining the probability of recurrent events. Here we shall derive this formula which is $f(t) = \sum_{N=1}^{\infty}(1/(2N-1))g(N,\lambda_1\lambda_2,+)$ where λ_1,λ_2

are the probabilities at the time t. Now if $\lambda_1 = \lambda_2$ the events are certain; when $\lambda_1 \neq \lambda_2$ the events are uncertain. Thus if we have a series of events Sx, where the elements of S succeed each other with the above mentioned probabilities we can establish a criterion for inductive laws. This can be accomplished by taking the elements of S in pairs and treating them as two events and so on. If the difference between λ_1 and λ_2 is small so is the uncertainty. Thus we could have a sequence of events where the probabilities differ by a small amount and appear to recur fairly regularly. (Received February 13, 1958.)

546-47. R. H. Bing: A 2-manifold-with-boundary in E^3 is tame if its complement is 1-ULC.

This is an extension of the result given earlier (page 8 of December 1957 Notices Amer. Math. Soc.) to the effect that a 2-manifold M' in E^3 is tame if $E^3 - M'$ is locally simply connected at each point of M'. Let M be a 2-manifold-with-boundary in E^3 such that $E^3 - M$ is locally simply connected at each point of M and D be a disk in M. It is shown that $E^3 - D$ is 1-ULC. It follows from the above mentioned result that D is locally tame at each point of Int D. By "splitting" D, we obtain a 2-sphere S containing D that is locally polyhedral mod D. Then $E^3 - S$ is 1-ULC and by the above mentioned result is tame. Since each disk D in M is tame, so is M. (Received February 28, 1958.)

546-48. G. H. Pimbley: A proof of existence and uniqueness for the solution of a discrete nonlinear boundary value problem.

The paper deals with solutions of the nonlinear system $Au + d(u) = 0$, where A is a symmetric N X N matrix of real elements a_{ij} such that the row sums vanish $\sum_{j=1}^{N} a_{ij} = 0$, $i = 1,\ldots,N$. Hence, the null space of A consists of vectors with equal components. u is the unknown vector with N components, u_i, $i = 1,\ldots,N$. d(u) is a vector of function components $d_i(u_i) = P_i + \delta(i,\mathcal{P}) f_i(u_i)$, $i = 1,\ldots,N$, where $P_i > 0$, $\mathcal{P}$ is a proper subset of the first N positive integers, $\delta(i,\mathcal{P}) = \{1, i \in \mathcal{P};\ 0, i \notin \mathcal{P}\}$, f_i is monotone decreasing, continuous, and such that $f_i(x) \to \infty$ as $x \to -\infty$, $f_i(x) \to -\infty$ as $x \to \infty$. This is the discrete version of the problem of steady state linear heat flow in solid material with convective cooling on the entire boundary. Here there is a nonlinear normal derivative condition on the boundary; the unknown is not prescribed anywhere. Methods used by G. Birkhoff and J. B. Diaz (Nonlinear network problems, Quart. App. Math. vol. 13, No. 4, Jan. 1956, pp. 431-443) are appropriately modified to prove

existence and uniqueness for this finite difference problem. The proof of existence is constructive. (Received February 28, 1958.)

546-49. G. H. Pimbley: Solution of the initial value problem for the linearized multi-velocity transport equation with a slab geometry.

The problem is as follows: To solve $\partial N/\partial t = AN$ for N, where $N(x,\mu,t)$ is an n-component vector, $A = -\mu V \partial/\partial x \cdot -\sigma V \cdot + 2^{-1} VC \int_{-1}^{+1} \cdot d\mu$; C is an $n \times n$ matrix of non-negative elements, and σ and V are diagonal positive matrices. The problem includes boundary conditions: $N(a,\mu,t) = 0, -1 \leqq \mu < 0$, $N(-a,\mu,t) = 0, 1 \geqq \mu > 0$, and an initial condition: $N(x,\mu,0) = f(x,\mu)$, $-a \leqq x \leqq a$, $-1 \leqq u \leqq$ where $f \in D(A)$ is a given n-component vector. Using L_2 product space, the spectral plane of A is discussed. A certain open right half plane $\mathcal{T}$ is such that $\mathcal{T} = \rho(A) \cup S$, where $S \subset P\sigma A$. S is closed and bounded. The remaining left half plane belongs to σA. The resolvent of A has properties such that the Hille-Yosida theorem can be applied. Hence, there exists a family of bounded operators which uniquely solves the initial value problem in the strong sense. If $D^{-1}CD$ is symmetric, where D is diagonal, or if C is triangular, then S is real and has only finitely many points. With C so restricted, the solution is presented as a finite expansion in eigenelements of A plus a residual term of lower order. (Received February 28, 1958.)

546-50. H. S. Shapiro: A power series with small partial sums.

Let $a_0, a_1, a_2, \ldots$ denote the successive terms of the sequence 1,1,1,-1,1,1, -1,1,1,1,1,-1,-1,-1,1,-1,... whose law of formation is as follows: $a_0 = a_1 = 1$; and given the first 2^k terms of the sequence ($k \geqq 1$) the next 2^k are obtained by copying the first 2^{k-1} followed by the negatives of the next 2^{k-1}. Let $f(z) = a_0 + a_1 z + a_2 z^2 \ldots (|z| < 1)$ and let $s_n(z) = \sum_{k=0}^{n} a_k z^k$. Then, for $|z| \leqq 1$ (i) $|s_n(z)| \leqq (2(n+1))^{1/2}$ when $m = 1,3,7,\ldots,2^k-1,\ldots$, (ii) $|s_n(z)| \leqq A(n+1)^{1/2}$ for all n, where A is a certain numerical constant. (Received March 3, 1958.)

546-51. H. S. Shapiro: Approximation by rational functions.

The following theorem is proved: Let Z be an enumerable set of complex numbers. A necessary and sufficient condition that every continuous function on $|z| = 1$ can be uniformly approximated by rational functions all of whose finite poles lie in the set Z is that both of the following series: $\sum_{z \in Z; |z|<1} (1-|z|)$ and $\sum_{z \in Z; |z|>1} (|z|-1)$ diverge. Here points of Z are

counted with multiplicities, the occurrence of a point α of Z with multiplicity k meaning that a pole of order k at α is permitted. Extensions to more general contours than circles are also given. (Received March 3, 1958.)

546-52. Donald Bratton: A theorem on factorization in semigroups that results from a theorem of Kleene's in the theory of automata.

The subsets of the free semigroups $\mathcal{S}(A)$ over a finite set A are called the events (over A). An event E is called regular (Kleene) when it can be constructed from the events $\{\{a\}\}$ ($a \in A$) by a finite number of the operations: $(E,F) \to E \cup F$, $(E,F) \to EF$, $E \to E^{\infty}$. Kleene's theorem asserts that an event E is regular if and only if there exists an automaton M with input set A and output $\{0,1\}$ which puts out 1 precisely when the input sequence so far is a word belonging to E. This theorem can easily be translated into the following equivalent result: The set of all factorizations $g = x_1 x_2 \ldots x_k$ of an element g of a finite semigroup G constitutes a regular event over G. (Received March 4, 1958.)

546-53. Donald Bratton: A machine process for maximizing a function of one real variable.

Given a continuous function f on a compact interval I and a Lipschitz condition $|f(x) - f(y)| \leqq K|x - y|$, the following algorithm, practicable for a high-speed automatic computer, computes a point $\hat{x}$ where f achieves its supremum over I. In each step f is observed at a strategically chosen point. Having observed $f(x_i) = v_i$ $(1 \leqq i \leqq n)$, f is majored in I by the polygonal function $E(x) = \min_i\{v_i + K|x - x_i|\}$. E(x) is stored in such a way that it can easily be maximized. Its maximizer is taken as the next point of observation. The machine stores the point $\hat{x}$ where the highest value of f has been observed so far. $\hat{x}$ always converges to a correct answer. (Received March 4, 1958.)

546-54. Donald Bratton: Concrete representation of the axiom of choice within Boolean algebra.

Boolean algebra can be formulated as a logical, quantified, and equalitative theory $\mathcal{T}$ (in the sense of Bourbaki) with the specific signs 0,1, =, and the axioms: $(\forall x)(x = 0)$ or $(x = 1)$; $0 \neq 1$. The usual duality between the terms and relations of Boolean algebra results. In addition, one has the criterion: If P is a term and x a letter, and Q denotes $(\Pi x)P$, the relation $((\exists x)(P = 1))$

$\longleftrightarrow$(((Q x)P) = 1) is a theorem. One is thus led to introduce in $\mathfrak{B}$ the following schema: Let R be a relation, T the term associated to R, x a letter; the relation $\tau_x(R) = (1|x)T$ is an axiom. This yields a concrete representation of the Axiom of Choice within $\mathfrak{B}$. (Received March 4, 1958.)

546-55. Donald Bratton: A fundamental theorem in the theory of continued fractions.

Let K be a field, $K \cup \{\infty\}$ the set obtained by adjoining ∞ with the (not everywhere defined) laws: $k + \infty = \infty$ $(k \neq \infty)$; $k \cdot \infty = \infty$ $(k \neq 0)$; $(\infty)^{-1} = 0$; $0^{-1} = \infty$. An arithmetic expression is called computable if the indicated operations can be carried out. Let (1) $a_0 + \underline{b_1|}\ a_1 + \underline{b_2|}\ \cdots + \underline{b_n|}\ a_n$ be a continued fraction whose terms $a_i, b_i \in K$; and P,Q be the last numerator and denominator convergents associated. The fraction (1) is computable if and only if $Q^{-1}P$ is computable, and then their values coincide. This result can be used to great advantage to establish the fundamental properties of continued fractions. It also clarifies some hitherto not well understood difficulties. (Received March 4, 1958.)

546-56. T. T. Frankel: Fixed points and torsion on Kähler manifolds.

Let M be a compact Kähler manifold, Φ a toral group operating holomorphically on M, F the fixed point set of Φ on M with components $\{F_\alpha\}$, and finally let $b_i(X)$ denote the ith Betti number of X. Theorem: If $b_i(M) = 0$ then F is not empty, and if $F \neq M$ then F is not connected. Theorem: If F is not empty there are even integers $\{\lambda_\alpha\}$ such that $b_i(M) \leqq \sum_\alpha b_{i-\lambda_\alpha}(F_\alpha)$. Theorem: If F is not empty and if $H_{2i+1}(F,Z) = 0$, all i, then $H_i(M,Z) = \sum_\alpha H_{i-\lambda_\alpha}(F_\alpha,Z)$ and M has no torsion. These generalize the result of Bott: If G is a compact Lie group and C(T') is the centralizer of a toral subgroup T' of G, then G/C(T') has no torsion and its odd dimensional Betti numbers vanish. (Received March 4, 1948.)

546-57. Larry Shepp and M. S. Klamkin: On least square circles and spheres.

Apparently, there does not seem to be much occasion to fit a least square circle, especially in comparison to fitting least square straight lines. Recently however, this problem came up in a practical design problem. Since a search through many of the standard texts covering least square fitting failed to un-

cover a solution to this problem, we present the following elementary solution: Starting with a set of N points (x_n,y_n) the natural circle to consider would be the one which minimizes the sum of the squares of the distances between (x_n,y_n) and the circle. This leads to a set of three simultaneous equations which, apparently, cannot be solved analytically. However, if we minimize $\sum\{(x_n - h)^2 + (y_n-k)^2 - r^2\}^2$, we are led to a set of three linear equations which have a unique solution. We also obtain a least square sphere in a similar fashion. (Received March 5, 1958.)

546-58. Avron Douglis: On discontinuous initial value problems for certain nonlinear partial differential equations.

Consider a partial differential equation of the form (*) $u_t + (F(u))_x = 0$ with uniformly convex, continuously differentiable F(u). The problem of finding a solution u(x,t) of (*) with prescribed bounded, measurable initial data $\bar{u}(x)$ such that $\bar{u} = 0$ for $x < 0$ here is interpreted as that of producing a Lipschitz-continuous function U(x,t) such that $U(x,0) = \int_0^x \bar{u}(x')dx'$ and that $U_t + F(U_x)-F(0) = 0$ almost everywhere. (Slightly different generalizations have been used previously by Lax, Oleinik, Vvedenskaya and the author.) This problem is solved with a function U(x,t) such that, whenever $t > 0$ and $U_x(x,t)$ and $U_x(x't)$ exist, (**) $(U_x(x,t)-U_x(x',t))/(x - x') \leq t^{-1}$. A mixed difference-differential technique is employed which, it is hoped, may prove useful with more general types of equations. (Received March 4, 1958.)

546-59. W. G. Bade: Similarity equivalence of spectral operators.

Let T be a scalar type spectral operator in $\mathcal{X}$ in the sense of Dunford (Pacific J. Math. vol. 4 (1954) pp. 321-355). As a byproduct of the methods of the preceding abstract we have the following theorem: If the resolution of the identity E(·) of T contains no projections of infinite uniform multiplicity, then T is similarity equivalent, $T = A^{-1}NA$, to a normal operator N on an appropriate Hilbert space where the operator $A: \mathcal{X} \to \mathcal{Y}$ and its inverse are closed and densely defined. Moreover, A sets up a one to one correspondence between the closed subspaces of $\mathcal{X}$ invariant under E(·) and those in $\mathcal{Y}$ invariant under the resolution of the identity of N. (Received April 3, 1958.)

546-60. W. G. Bade: Multiplicity of spectral measures. II.

Let $\mathcal{L}$ be a complete Boolean algebra (B.A.) of projections in a Banach space $\mathcal{X}$ and $\mathcal{L}^*$ be the B. A. of adjoints in $\mathcal{X}^*$. For $x \in \mathcal{X}$, $x^* \in \mathcal{X}^*$, define $\mathcal{M}(x) = \text{clm}\{Ex | E \in \mathcal{L}\}$, $\mathcal{M}(x^*) = \text{weak}^*\text{clm}\{E^*x^* | E^* \in \mathcal{L}^*\}$. Suppose $\mathcal{L}$ satisfies the countable chain condition for simplicity. Then the multiplicity $m(E)$ of E in $\mathcal{L}$ is the smallest cardinal of a set of subspaces $\mathcal{M}(x)$ spanning $E\mathcal{X}$. Similarly we define $m(E^*)$ using the weak* topology. Extending work of Dieudonné it was shown (Bull. Amer. Math. Soc. Abstract 63-4-556) that the identity I in $\mathcal{L}$ may be decomposed into the sum of projections E of uniform multiplicity i.e. such that $m(F) = m(E)$ whenever $0 \neq F \leq E$. This result is now extended to the adjoint B.A. $\mathcal{L}^*$. The following duality theorem is proved: A projection E in $\mathcal{L}$ has finite multiplicity n iff its adjoint E^* in $\mathcal{L}^*$ has multiplicity n. (Received April 3, 1958.)

546-61. William Browder: Loop spaces of suspensions. I.

Kudo and Araki (Mem. Fac. Sci., Kyūsyū Univ. vol. X, no. 2 (1956)) have defined the concept of an H_n-space, and used it to define homology operations mod 2 in iterated loop spaces. If Q is an H_n-space, one can define an operation of two variables $\psi_n(x,y)$ where $\psi_n(x,y) \in H_{p+q+n}(Q;A)$ if $x \in H_p(Q;A)$, $y \in H_q(Q;A)$, where A is any coefficient domain. If Q is an H_0-space (an H-space in the usual sense) $\psi_0(x,y) = x*y - (-1)^{pq}y*x$, where $*$ denotes the Pontrjagin product. The n-th loop space $\Omega^n(X)$ of an H-space X is an H_n-space. Let σ denote the homology suspension associated with the acyclic fibre space $E(\Omega^{n-1}(X)) =$ the space of pathes of $\Omega^{n-1}(X)$, with fibre $\Omega^n(X)$. Then σ: $H_k(\Omega^n(X);A) \to H_{k+1}(\Omega^{n-1}(X);A)$. Theorem: $\sigma\psi_n(x,y) = \psi_{n-1}(\sigma x,\sigma y)$ or in other words, $\psi_{n-1}(s,t)$ is transgressive if s and t are. Using this result and results of Kudo and Araki one can generalize their results on $H_*(\Omega^k(S^n);Z_2)$ ($k <$ to calculate $H_*(\Omega^n(s^nY);Z_2)$, the homology ring of the n-th loop space of an n-fold suspension of an arcwise connected space Y, in terms of the homology of Y, coefficients Z_2(the integers modulo 2). (Received April 17, 1958.)

546-62. T. C. Chen: A modified power method for finding the eigenvalues and vectors of matrices. Preliminary report.

In the power method an entirely new vector replaces the old one after an iteration although vector elements are generated one at a time usually. The procedure can be modified such that new vector elements, properly scaled, an

directly incorporated into the old vector. This has merit of simplicity in machine programming and in all the cases tried leads to superior convergence. (Received March 17, 1958.)

546-63. Albert Edrei and W. H. J. Fuchs: A sharpened form of Picard's theorem for meromorphic functions of finite order.

Let $f(z)$ be a meromorphic function of order λ. Put $K(f) = \lim \sup_{r\to\infty}\{(N(r,0) + N(r,\infty))/T(r)\}$ where $N(r,a)$ and $T(r)$ are the standard symbols of Nevanlinna's theory and let $K_1(\lambda) = \inf K(f)$ where f ranges over all meromorphic functions of order λ. Improving previous results of Nevanlinna, Shah and Goldberg, the authors prove: (i) $K_1(\lambda) = 1$ for $0 \leqq \lambda \leqq 1/2$; (ii) $K_1(\lambda) = \sin(\pi\lambda)$ for $1/2 < \lambda \leqq 1$; (iii) $K_2(\lambda) = \{|\sin\pi\lambda|/(2.2\,\lambda+|\sin\pi\lambda|/2)\} \leqq K_1(\lambda) \leqq (3.3)K_2(\lambda)$ for $1 < \lambda < +\infty$. (Received March 6, 1958.)

546-64. Martin Greendlinger: Dehn's algorithm for the word problem.

Let the group G be defined by generators and relations $R_i = 1$. The words R_i on the generators are called relators. Include in the set of relators any word arising from $R_i^{\pm 1}$ by cyclic permutation of the symbols. Assume that in the product of two relators of the extended system one sixth or more of one relator can be canceled only if the two relators are inverses. Then the word-problem in G can be solved on account of the following theorem: If a word W in the generators has the property $W = 1$ in G, then it contains more than 1/2 of at least one of the relators of the extended system. This solution was given by Dehn for the fundamental groups of closed surfaces of a genus > 1. (Received March 10, 1958.)

546-65. F. B. Jones: On the existence of weak cut points in plane continua.

A general theorem is proved from which it follows as a corollary that: If p and q are points of different components of the set of interior points of a bounded plane continuum M and M does not separate the plane, then there exists a point x such that every subcontinuum of M which contains both p and q must also contain x. (Received March 14, 1958.)

546-66. W. J. Klimczak: The domain of absolute convergence of certain eigenfunction series.

The series considered are of the type (1) $f(z) = \sum_{n=0}^{\infty} f_n w_n(z)$, where the

$w_n(z)$ are eigenfunctions of the differential operator $-d^2/dz^2 + z^N$ with N a positive integer. The relationship between N and R(N), the domain in the complex plane in which series (1) converges absolutely, is determined. Estimates of the asymptotic behavior for large values of n of the functions $w_n(z)$ and the corresponding eigenvalues are obtained. It is found that R(N) consists of the strip $-\tau < \mathrm{Im}(z)$ such that $\tau = \lim\sup_{n\to\infty}(n^{-CN/(N+2)})|f_n|$. Here C is a positive constant which depends only upon N. The eigenfunctions $w_n(x)$ form a complete orthonormal set in $L^2(0,\infty)$. The case N = 1, in which the $w_n(x)$ are constant multiples of translates of a fixed function in $L^2(0,\infty)$, is of special interest. (Received March 6, 1958.)

546-67. Walter Littman: Finite difference methods for elliptic equations of arbitrary order.

The first existence proof for the Dirichlet problem for a second order elliptic equation and the biharmonic equation was given for the case of two independent variables by Courant, Friedrichs and Lewy (Math. Ann. vol. 100 (1928)). In the present paper the Dirichlet problem for $(L + \lambda)u = f$ is treated by finite differences. Here L is an elliptic operator of order 2m in a bounded domain D in n-space. For λ sufficiently large the following is proved: For any $f \in L_2(D)$ a sequence of finite difference problems can be solved which converge to a generalized solution u assuming boundary values in a generalized sense. Well-known differentiability theorems can then be applied; for example, if f is sufficiently regular, u will be a strict solution. The proof relies on a finite-difference analog of Gårding's inequality. (Received March 17, 1958.)

546-68. H. E. Salzer: A mixed-type best approximation problem.

For an overdetermined n by m system of linear equations $Ax = b$, $A \equiv \|a_{ij}\|$ of rank m, $r_i = \sum_{j=1}^{m} a_{ij}x_j - b_i$, $i = 1,2,\ldots,n$, $x \equiv (x_1,x_2,\ldots,x_m)$, $b \equiv (b_1,b_2,\ldots,b_n)$ many practical problems might tolerate a $\max|r_i| = R$, $M < R < L$, where $M = \max|r_i|$ for x_M minimizing $\max|r_i|$, and $L = \max|r_i|$ for x_L minimizing $\sum_{i=1}^{n} r_i^2$. The residually bounded least squares solution (r.b.l.s.s.) x_R minimizes $\sum_{i=1}^{n} r_i^2$ subject to $|r_i| \leqq R$ (a special problem in quadratic programming). A finite algorithm obtains the r.b.l.s.s. by a method of successive reductions from an n by m system to an (n-1) by (m-1) system, and the employment of a finite algorithm for $m = 1$. The proof is based upon the fact that $S_R = \min\sum_{i=1}^{n} r_i^2$ is a strictly decreasing function of R for $M < R < L$. This mono-

tonicity property serves to solve also the inverse problem, i.e. for preassigned $S = \sum_{i=1}^{n} r_i^2$ to find x_D minimizing $D = \max|r_i|$, the solution x_D giving $D =$ that R for which $S_R = S$, so that $x_D =$ the corresponding x_R. For the r.b.l.s.s. when n is large or infinite (a typical problem in this latter continuous case being to determine the coefficients a_i, $i = 0, 1, \ldots, m-1$, in the polynomials $R_m(x) = x^m + \sum_{i=0}^{m-1} a_i x^i$ "between" Chebyshev and Legendre polynomials, which minimize $\int_{-1}^{1} [\sum R_m(x)]^2 dx$ subject to $|R_m(x)| \leqq R$, $-1 \leqq x \leqq 1$), a method of linear inverse interpolation between x_M and x_L might yield a first approximation to x_R (subject to certain limitations). (Received April 17, 1958.)

546-69. C. C. Faith: Concerning noninvariant submodules of simple, matrix, and local matrix rings.

A denotes a simple algebra over a field F; A contains 1, and an idempotent $e \neq 1$; dimension $A/F \neq 4$ when F has characteristic $c(F) = 2$. Let S be a subspace, $S \not\supseteq [A,A]$ and $S \not\subseteq Z$, the center of A (c f., Amitsur [1, Theorem 1]). (I): If F is infinite, then (1) there is a $u \in A$, $u^2 = 0$, such that $\{(1 + au)^{-1} \cdot S(1 + au) | a \in F\}$ is infinite. (II) If $c(F) = 0$, then (2) for some $u \in A$, $u^2 = 0$, different k determine different subspaces $(1 + u)^{-k} S(1 + u)^k$. (III): If P is a proper subalgebra, $P \not\subseteq Z$, then (1) and (2) hold with $S = P$ (c f.,[1, Theorem 2]). (I)-(III) may be restated when $1 \notin A$. From now on F is an arbitrary ring with 1. Similar results are noted for the matrix rings $A = F_n$, and local matrix rings of Rosenberg [2], e.g.: Let S be an additive subgroup of F_n, $n \geqq 3$, not of the form $[F_n, I_n] + D$ of [2, Theorem 1] and $S \not\subseteq C$, the center of F_n. (IV): If the additive group F is divisible, then (1) holds for some $u = e_{ij}$, $i \neq j$. (V): If S is divisible, then (2) holds for some $u = ae_{ij}$, $a \in F$, $i \neq j$. (VI): If F is simple; $c(F) = 0$; R a proper subring of F_n, $R \not\subseteq C$, then $\{y^{-1}Ry | y \in F_n\}$ is infinite. [1] Proc. Amer. Math. Soc. vol. 7 (1956) pp. 987-989. [2] Ibid pp. 891-898. (Received May 8, 1958.)

THE JUNE MEETING IN CORVALLIS, OREGON

June 20, 1958

547-1. R. S. Pierce: Translation lattices. I.

A translation semi-lattice (or t.s.l.) is the abstract analogue of a system of bounded, real valued functions (on some set) which is closed under addition of constants and pointwise g.l.b. operation. Theorem. If T is a t.s.l. and $\phi \neq I \subseteq T$ is uniformly closed, bounded and satisfies $f \leqq g \in I$ implies $f \in I$, then there is a uniquely determined triple (X, C, Δ) consisting of a compact Hausdorff space X, a point separating t.s.l. $C \subseteq C(X)$ such that every nonempty open set of X contains some nonempty set of the form $[x|f(x) > 0]$ where $f \in C$, and finally a homomorphism Δ of T on C such that $\Delta^{-1}[f \in C | f \leqq 0] = I$. The triple (X, C, Δ) is called a representation of the pair (T,I). Theorem. If (X, C, Δ) and $(\overline{X}, \overline{C}, \overline{\Delta})$ are representations of (T,I) and $(\overline{T}, \overline{I})$ respectively, and if Φ is a homomorphism of T onto $\overline{T}$ satisfying $\Phi^{-1}(\overline{I}) = I$, then there is a closed subset $\overline{Y}$ in $\overline{X}$ and a continuous mapping π of $\overline{Y}$ onto X such that $[\Delta(f)](\pi \overline{y}) = [\overline{\Delta}(\Phi f)](\overline{y})$ for all $f \in T$ and $\overline{y} \in \overline{Y}$. These results are used to investigate representations by continuous functions of such systems as the upper semi-continuous, lower semi-continuous and normal lower semi-continuous (bounded) functions on a topological space. (Received April 2, 1958.)

547-2. C. S. Coleman: Integral curves of a certain class of differential systems.

Consider the system (*) $dx/dt = f(x)$, where f is a real n-vector function whose components are homogeneous polynomials of degree $m \geqq 1$ in the components of the n-vector x. The homogeneity of the components of f(x) implies that whenever x(t) is a solution of (*) then so is $kx(k^{m-1}t)$, k constant. The surface S(x): $x = kx(t)$, $k > 0$, $-\infty < t < \infty$, is then an integral surface of (*). For $n = 3$ this leads to a classification of integral surfaces of (*) and to a classification (with one exception) of the integral curves of these surfaces. (Received April 9, 1958.)

547-3. J. H. Wells: Concerning the Hausdorff inclusion problem.

The formulation, due to Hille and Tamarkin [Proc. Nat. Acad. Sci. U. S. A. vol. 19 (1933) pp. 573-577] of the Hausdorff inclusion problem in terms of the

integral equation of R. Schmidt is extended, using a mean Stieltjes integral (for which there is no "exceptional set") to cover nonregular Hausdorff means $[H,\phi]$ where ϕ (not necessarily in B. V. $[0,1]$) has only discontinuities of the first kind. The moment problem for these functions ϕ is solved. Necessary and sufficient conditions for the regular Hausdorff mean $[H,\psi]$ to include the Cesaro mean (C,k), $k > 0$, are given in terms of convexity properties of ψ and derivatives (of fractional order, defined in a new way appropriate to the problem, if k is not an integer). (Received April 14, 1958.)

547-4. L. E. Ward, Jr.: Fixed point theorems for arcwise connected spaces.

The following theorem is proved. Let M be a separable metric arcwise connected space which contains no simple closed curve. Suppose that for each sequence x_n in M the sequence $[x_n,x_{n+1}]$ of arcs has a limit point. Then M has the fixed point property. The proof employs methods first used by J. L. Kelley and G. S. Young. With appropriate auxiliary hypotheses the theorem may be extended to include point-to-set mappings which are continuous in various senses. In fairly restricted situations one may then obtain a characterization of the fixed point property for continuous point-to-set mappings. (Received April 14, 1958.)

547-5. John Hilzman: Application of the Fréchet differential to the approximate solution of the Volterra integral equation.

The resolvent kernel $S[K]$ and the solution $y[K]$ of the Volterra integral equation of the second kind are treated as functionals (in the sense of A. D. Michal) of the given kernel K. The given kernel is replaced by an approximate kernel K_0, with known resolvent $S[K_0]$, yielding a solution $y[K_0]$. An approximation to the exact solution $y[K]$ is obtained from a Taylor-type expansion, in Fréchet differentials of $y[K]$ evaluated at $y[K_0]$ with increment $dK = K - K_0$, of $y[K]$: (1) $y[K] = y[K_0] + \sum_{j=1}^{\infty}(d^j y[K_0;dK]/j!)$. Precise error estimates are available in terms of norms. It is shown that for any n-order approximation from (1) $\| y[K] - y[K_0] - \sum_{j=1}^{n}(j!)^{-1} d^j y[K_0;dK]\| \leq \| y[K_0]\| \{e^{uh(1+uB)} - \sum_{j=0}^{n}(j!)^{-1}[uh(1+uB)]^j\}$ where $\| G(x,t)\| = \max_{a\leq t\leq x\leq b}|G(x,t)|$, $\|y(x)\| = \max_{a\leq x\leq b}|y(x)|$, $u = |b - a|$, $h = \|dK\|$, and $B = \| S[K_0]\|$. (Received April 16, 1958.)

547-6. H. H. Schaefer: On the Fredholm alternative in locally convex spaces.

By a different and uniform method, based on an extensive use of weak topology, this paper discusses the behavior of adjoints, and gives characterizations, of σ-maps and maps for which the Fredholm alternative or even Riesz' theory of compact maps hold. (A σ-map is a homomorphism T of one locally convex space E into another F, such that T(E) is closed in F and both the kernel and cokernel of T are finite dimensional.) Examples: 1. Let E,F be F-spaces. Then a linear map on E into F is Fredholm if and only if its strong adjoint is. 2. If U is a continuous linear map on E into E, and is such that $I - U^n$ (I identity) is a Riesz trf. for some n, then I - U is a Riesz trf. The paper is largely expository in the sense that many of its results, by various methods, have been proved for Banach spaces before this. The results are then used to show that the spectrum of a bounded map is bounded and closed, e.g. if E is a Fréchet-space. There is no discussion of maps I - U (I identity, U compact), whose theory is well-known even for locally convex spaces. (Received April 28, 1958.)

547-7. C. A. Swanson: Asymptotic perturbation series for characteristic value problems.

Let L denote an ordinary linear differential operator of order n on a half-open real interval (0,b] (b > 0) which contains no singular points. Suppose x = 0 is a regular singular point. Characteristic value problems associated with L are under consideration on a perturbed interval [ϵ,b] where ϵ is a small positive number. Under certain assumptions, the characteristic values λ_ϵ and the characteristic functions u_ϵ are developed in asymptotic series valid as $\epsilon \to 0$. The series are of a more general type than asymptotic power series. (Received April 28, 1958.)

547-8. M. D. Marcus and Rimhak Ree: Diagonals of doubly stochastic matrices.

Let S be a real n-square doubly stochastic (d.s.) matrix. Results: (i) If R = S'S and $g(t_1,\ldots,t_n)$ is convex in each t_j for $0 \leqq t_j \leqq 1$ then there exists a permutation σ of the integers 1,...,n such that $s_{i\sigma(i)} > 0$, i = 1,...,n, and $g(r_{11},\ldots,r_{nn}) \leqq g(s_{1\sigma(1)},\ldots,s_{n\sigma(n)})$. (ii) If S has k eigenvalues of absolute value 1 then there exists a permutation σ such that $s_{i\sigma(i)} > 0$, i = 1,...,n and (*) $\sum_{i=1}^{n} s_{i\sigma(i)} \geqq k$. The equality in (*) holds if and only if there exist permu-

tation matrices P, Q such that $PSQ = \sum_{j=1}^{k} \cdot n_j^{-1} J_{n_j}$, $\sum_{j=1}^{k} n_j = n$ where J_{n_j} is an n_j-square matrix each of whose entries is 1 and $\sum \cdot$ denotes direct sum of matrices. (iii) If $n = 2k$ ($n = 2k + 1$) then there exists a permutation σ such that $s_{i\sigma(i)} \geqq k^{-1}(k + 1)^{-1}$ ($s_{i\sigma(i)} \geqq (k + 1)^{-2}$), $i = 1,\ldots,n$. These bounds are best possible in the sense that there exist d.s. matrices in every dimension for which the equality holds for some i_0, $1 \leqq i_0 \leqq n$. (iv) Any d.s. matrix S is in the convex hull of at most $(n - 1)^2$ permutation matrices. The number $(n - 1)^2$ is best possible. Certain corollaries resulting from particular choices of the function g in (i) are stated. (Received April 29, 1958.)

547-9. A. E. Livingston and Lee Lorch: The zeros of some sine-integral-like functions.

If f(t) is sine-like in the sense that $f(t) \geqq 0$ and Lebesgue integrable on $[0,1]$, $f(t + n) = (-1)^n f(t)$ for positive integers n, and $f(t) = 0$ on at most a set of measure zero and if g(t) is completely monotonic on $(0,\infty)$ and $g(t) \to 0$ as $t \to \infty$, then the function $F(x) = \int_x^\infty f(t)g(t)dt$ has precisely one zero z_n in $(n,n + 1)$. The authors show that $z_n - n \geqq C$ for $n = 0,1,2,\ldots$, $\{z_n - n\}_0^\infty$ is decreasing, and $z_n - n \to p$ as $n \to \infty$, where C and p are determined by the conditions $0 < C < 1$, $2\int_0^C f(t)dt = \int_0^1 f(t)dt$, $0 < p < 1$, $\int_0^p f(t)h(t)dt = L\int_0^1 f(t)h(t)dt$, $h(t) = \lim_n g(t + n)/g(n)$, and $L = \lim_{\lambda \to \infty} [\sum_{k=0} (-1)^k g(\lambda + k)]/g(\lambda)$ (both of these latter limits existing). (Received May 5, 1958.)

547-10. Roy Leipnik: Linear integral equations for the generating functions of a class of functionals of Markov processes.

Let X be a vector Markov process from $\Omega \times T$ to V, and let $F_1,\ldots,F_q$ be a set of continuous functions from W^n to R, where $W = V \times T$. Let $L_1,\ldots,L_q$ be defined by $L_j(t_0,t) = \int_{t_0}^t \cdots \int_{t_0}^t F_j(X(t_1),t_1;\ldots X(t_n),t_n)dt_1 \ldots dt_n$ for $j = 1,\ldots,q$. Linear integral equations are derived for the conditional expectation of the joint generating function of $L_1,\ldots,L_q$, given $X(t_0)$ and $X(t)$. This is an extension of the work of Kac, Darling, and Siegert who considered $q = 1$, $n = 1$. The case $q = 2$, $n = 2$ is shown to have applications to the sampling distributions of estimators of the spectral density of Markov processes. Solutions are obtained by bi-orthonormal expansions for $n = 1$ and, under severe restrictions on X and the F_j, for $n = 2$. (Received May 5, 1958.)

547-11. Herman Rubin: On a problem of Kurepa concerning the axiom of choice.

The symbolism is that used by Kurepa (Bull. Soc. Math. France vol. 80, pp. 225-232), with $K^*F \leftrightarrow \overline{K}F \cap VF$. Kurepa asks for which sets S, $(F)(\bigcup\{\phi F: \phi \in S\})$ is equivalent to the axiom of choice. We show that the theorem is true for any of the following four sets or their subsets: $\{D,\bar{I},K\}$, $\{D,\bar{I},K^*\}$, $\{\bar{D},I,\bar{I},K\}$, $\{\bar{D},I,K^*\}$. If the maximal subfamily of F can be required to contain a given element, then all six of the conditions can be included in S. (Received May 5, 1958.)

547-12. B. L. Foster: Covariant differentiation.

In the paper Differentiation on manifolds without a connection, (to appear in the Michigan Journal of Mathematics) a generalized notion of tensor was introduced. If T is an n-times controvariant and m-times covariant tensor, then $(T, \partial_s T)$ is a generalized tensor. If Γ^a_{hs} is an affine connection, then the column "vector" with Γ^a_{hs} as its first entry and 1 as its second entry is a generalized tensor, denoted by γ; the column "vector" with $-\Gamma^a_{hs}$ as its first entry and 1 as its second entry is a generalized tensor, denoted by $\bar{\gamma}$. Products, which are again generalized tensors, are defined, e.g., $\gamma\bar{\gamma}$ has $\Gamma^a_{ks}\delta^h_b - \delta^a_k\Gamma^h_{bs}$ as its first entry and 1 as its second entry. Theorem: The covariant derivative of T is the ordinary tensor obtained by contracting $(T, \partial_s T)$ with $\gamma^n\bar{\gamma}^m$. A natural generalization of covariant differentiation is obtained by replacing $\gamma^n\bar{\gamma}^m$ by the most general tensor of its type, for each n, m. The deviation from commutativity of the covariant differentiation so defined is not in general expressible by a curvature tensor and a torsion tensor. Appropriate specialization of generalized tensors having the same type as $\gamma^n\bar{\gamma}^m$ yield the "tensor connections" of E. Bompiani, Le connessioni tensoriali, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. vol. 1 (1946) pp. 478-482. (Received May 6, 1958.)

547-13. R. F. Anderson and R. L. Blair: Spaces of real ideals.

Let X be a topological space, C(X) the algebra of all continuous functions on X to the real numbers R, and A a subalgebra of C(X) that contains the unit of C(X). Definition 1: A is point-determining in case an ideal M of A is real (i.e., $A/M \cong R$) if and only if $M = \{g \in A;\ g(x) = 0\}$ for some unique $x \in X$; and A is pseudo-regular in case whenever $x \in X$ and U is an open neighborhood of x there is a $g \in A$ such that $g(x) = 0$ and $g \geq 1$ on $X - U$. Definition 2:

If $S \subseteq C(X)$ and $x \in X$, then S is transcendental over A at x in case whenever $f_1,\ldots,f_n \in S$ and $P(f_1,\ldots,f_n)$ is a polynomial in $f_1,\ldots,f_n$ with coefficients in A, $P(f_1,\ldots,f_n)=0$ implies that each coefficient of P vanishes at x; set $T(S) = \{x \in X;$ S is transcendental over A at $x\}$. Theorem: If A is point-determining and pseudo-regular, if $S \subseteq C(X)$ with $T(S) = X$, and if B is the subalgebra of C(X) generated by A and S, then the space of real maximal ideals of B (in the weak topology induced by B) is homeomorphic to a product space $X \times R^{|S|}$ where $|S| =$ card S. Generalizations are considered for the case in which T(S) is not necessarily X; in this case the space of real ideals of B is homeomorphic to a subspace of $X \quad R^{|S|}$. (Received May 7, 1958.)

547-14. F. W. Anderson: A class of function algebras.

In order to characterize the algebra C(X) of all continuous real-valued functions on a completely regular space X, the author and R. L. Blair, in an earlier work (cf. these Notices, Abstract 539-39 vol. 5, p. 71) first characterized a certain large class $\mathcal{A}$ of subalgebras A of C(X) and then obtained conditions which insure that such an algebra A is all of C(X). In this paper it is shown that if X is a P-space (see Gillman and Henriksen, Trans. Amer. Math. Soc. vol. 77), then the selection from $\mathcal{A}$ of the algebra C(X) can be effected more easily than in the general case. The primary reason for this state of affairs is that if X is a P-space, then (i) the "zero-sets" of X are associated, in an obvious fashion, with certain easily described ideals of the algebras in $\mathcal{A}$, and (ii) the algebra C(X) is closed under certain infinite operations. Finally, characterizations of C(X), for X a P-space, not only as an algebra but also as an ℓ-ring and as a vector lattice, are obtained. (Received May 7, 1958.)

547-15. I. L. Glicksberg: Convolution semigroups of measures.

Let S be a compact abelian semigroup (c.a.s.) $\tilde{S}$ the set of normalized non-negative regular Borel measures on S, and C(S) the Banach space of complex valued continuous functions on S. Under convolution and the w* topology of $C(S)^*$, $\tilde{S}$ forms a c.a.s. Modifying the approach of Wendel (Proc. Amer. Math. Soc. vol. 5 (1954) pp. 923-928), the fact that the least (closed) ideal in a c.a.s. is a group is applied to $\tilde{S}$ to determine its subgroups: each subgroup $\tilde{G}$ of $\tilde{S}$ consists of all G-translates of Haar measure on $g \subset G$, where g,G are subgroups of S. A consequence is an extension of the Weyl equidistribution theorem: for $\mu \in \tilde{S}$, $N^{-1}\sum_1^N \mu^n \rightarrow$ Haar measure on the least ideal of the closed subsemigroup

of S generated by the carrier of μ. Similarly the subgroups of the convolution semigroup formed by the unit ball of $C(S)^*$ are determined; the results can be applied to the ball of $C_0(G)^*$, G a locally compact abelian group, by virtue of a result of Eberlein (Duke Math. J. vol 22 (1955) pp. 465-468). (Received May 7, 1958.)

547-16. Morgan Ward: A diophantine problem associated with the homogeneous product sum.

The homogeneous product sum considered is the symmetric function h_n of weight n on three indeterminates a,b,c; $h_n = \sum a^r b^s c^t$. Here $r + s + t = n$ and $0 \leqq r,s,t \leqq n$. The diophantine problem is whether h_n can vanish for $n > 1$ and a,b,c integers. Only odd n need be considered. The simplest case $n = 3$ reduces to the question of the existence of rational points on a certain cubic $y^2 = 4x^3 + g_2x + g_3$ where g_2, g_3 are large integers. In general the question reduces to whether a trinomial equation $z^m + pz + q = 0$ with $m = n + 2$ and p,q co-prime integers can have three integral roots. (Received May 7, 1958.)

547-17. Evelyn Frank: A new class of continued fraction expansions for the ratios of hypergeometric functions. II

The author continues the study of continued fraction expansions for the ratios of contiguous hypergeometric functions (cf. Trans. Amer. Math. Soc. vol. 87 (1956) pp. 453-476). The relations between these and well-known continued fractions are studied. New expansions are obtained. Recurrence formulas are found for the denominators of the approximants. (Received May 7, 1958.)

547-18. E. C. Riggle: An alternating gradient method for the solution of linear systems.

To solve a system of linear equations $Ax - b = 0$ where A is any $m \times n$ matrix and x,b are vectors, by a method of alternating gradients of the function $f(x) = |(Ax - b)_1| + \ldots + |(Ax - b)_m|$. The following iteration method is proposed: $Ax_i - b = r_i$, grad $f_i = A^T \operatorname{sgn} r_i, e_{i,j} = -\operatorname{grad} f_i^T v_j / v_j^T v_j$ $(j = 0,1,\ldots,i-1)$, $\mu_i = \operatorname{sgn} r_i + e_{i,i-1}\mu_{i-1} + \ldots + e_{i,0}\mu_0, v_i = A^T\mu_i, \lambda_i = r^T\mu_i / v_i^T v_i, x_{i+1} = x_i - \lambda_i v_i$. This method gives an exact solution, if one exists, in N mn (m,n) iterations. The test for inconsistency is the vanishing of the vector v_i when $r_i \neq 0$. Thus, the linear combination demonstrating the inconsistency is developed. (Received May 7, 1958.)

547-19. J. E. McFarland: An iterative solution of the quadratic equation in Banach space.

Let X be a Banach space, y an element of X, A a linear operator in X, and B a bilinear operator in X. Solution of the "quadratic" equation (1) $Ax + Bxx = y$ is attempted by the iteration $x_{n+1} = (A + Bx_n)^{-1}y$, $x_0 \in X$. If $x_n \to x \in X$, then x is shown to be a solution of (1). Sufficient conditions are given for the existence of $(A + Bx_n)^{-1}$, $n = 0,1,\ldots$, and for the convergence of the sequence $\{x_n\}$ to an element $x \in X$. The convergence proof yields an error bound for $\|x - x_n\|$. The given method is applicable to the numerical solution of an integral equation of Chandrasekhar and Ambarzumian. (Research performed under terms of Contract No. DA-04-200-ORD-177 between the Office of Ordnance Research, U. S. Army, and the Oregon State Board of Higher Education). (Received May 7, 1958.)